Contents

Preface **xi**

Acknowledgements **xv**

Navigating this Book **xvii**

I Foundations

1 ***Superposition, Entanglement and Reversibility*** **3**

2 ***A Brief History of Quantum Computing*** **11**

3 ***Qubits, Operators and Measurement*** **17**

3.1 Quantum Operators **22**

Unary Operators **22**

Binary Operators **26**

Ternary Operators **28**

3.2 Comparison with Classical Gates **30**

3.3 Universality of Quantum Operators **31**

3.4 Gottesman-Knill and Solovay-Kitaev **31**

3.5 The Bloch Sphere **32**

3.6 The Measurement Postulate **33**

3.7 Computation-in-Place **35**

4 ***Complexity Theory*** **37**

4.1 Problems vs. Algorithms **37**

4.2 Time Complexity **38**

4.3 Complexity Classes **40**

4.4 Quantum Computing and the Church-Turing Thesis **43**

II Hardware and Applications

5 *Building a Quantum Computer* 47

5.1 Assessing a Quantum Computer **47**

5.2 Neutral Atom **49**

5.3 NMR **50**

5.4 NV Center-in-Diamond **51**

5.5 Photonics **52**

5.6 Spin Qubits **54**

5.7 Superconducting Qubits **56**

5.8 Topological Quantum Computation **57**

5.9 Trapped Ion **58**

5.10 Summary **59**

6 *Development Libraries for Quantum Computer Programming* 61

6.1 Quantum Computers and QC Simulators **62**

6.2 Cirq **64**

6.3 Qiskit **66**

6.4 Forest **69**

6.5 Quantum Development Kit **71**

6.6 Dev Libraries Summary **74**

Using the Libraries **75**

Other Development Libraries **75**

6.7 Additional Quantum Programs **76**

Bell States **76**

Gates with Parameters **77**

7 *Teleportation, Superdense Coding and Bell's Inequality* 81

7.1 Quantum Teleportation **81**

7.2 Superdense Coding **84**

7.3 Code for Quantum Teleportation and Superdense Communication **85**

7.4 Bell Inequality Test **88**

8 *The Canon: Code Walkthroughs* 95

8.1 The Deutsch-Jozsa Algorithm **97**

8.2 The Bernstein-Vazirani Algorithm **104**

8.3 Simon's Problem **107**

8.4 Quantum Fourier Transform **108**

8.5 Shor's Algorithm **111**

RSA Cryptography **111**

The Period of a Function **113**

Period of a Function as an Input to a Factorization Algorithm **114**

8.6 Grover's Search Algorithm **126**

9 *Quantum Computing Methods* 131

9.1 Variational Quantum Eigensolver **131**

VQE with Noise **136**

More Sophisticated Ansatzes **138**

9.2 Quantum Chemistry **139**

9.3 Quantum Approximate Optimization Algorithm (QAOA) **144**

Example Implementation of QAOA **147**

9.4 Machine Learning on Quantum Processors **154**

9.5 Quantum Phase Estimation **160**

Implemention of QPE **163**

9.6 Solving Linear Systems **166**
Description of the HHL Algorithm **168**
Example Implementation of the HHL Algorithm **170**
9.7 Quantum Random Number Generator **178**
9.8 Quantum Walks **180**
Implementation of a Quantum Walk **182**
9.9 Summary **187**

10 ***Applications and Quantum Supremacy*** **189**
10.1 Applications **189**
Quantum Simulation and Chemistry **189**
Sampling from Probability Distributions **190**
Linear Algebra Speedup with Quantum Computers **190**
Optimization **190**
Tensor Networks **190**
10.2 Quantum Supremacy **190**
Random Circuit Sampling **191**
Other Problems for Demonstrating Quantum Supremacy **195**
Quantum Advantage **196**
10.3 Future Directions **196**
Quantum Error Correction **196**
Doing Physics with Quantum Computers **197**

III Toolkit

11 ***Mathematical Tools for Quantum Computing I*** **201**
11.1 Introduction and Self-Test **201**
11.2 Linear Algebra **203**
Vectors and Notation **203**
Basic Vector Operations **204**
The Norm of a Vector **208**
The Dot Product **211**

11.3 The Complex Numbers and the Inner Product **214**
Complex Numbers **214**
The Inner Product as a Refinement of the Dot Product **216**
The Polar Coordinate Representation of a Complex Number **220**
11.4 A First Look at Matrices **228**
Basic Matrix Operations **228**
The Identity Matrix **235**
Transpose, Conjugate and Trace **237**
Matrix Exponentiation **244**
11.5 The Outer Product and the Tensor Product **245**
The Outer Product as a Way of Building Matrices **245**
The Tensor Product **247**
11.6 Set Theory **250**
The Basics of Set Theory **250**
The Cartesian Product **253**
Relations and Functions **254**
Important Properties of Functions **259**
11.7 The Definition of a Linear Transformation **264**
11.8 How to Build a Vector Space From Scratch **266**
Groups **267**
Fields **273**
The Definition of a Vector Space **280**
Subspaces **282**
11.9 Span, Linear Independence, Bases and Dimension **285**
Span **285**
Linear Independence **287**
Bases and Dimension **289**
Orthonormal Bases **292**

12 *Mathematical Tools for Quantum Computing II* 295
12.1 Linear Transformations as Matrices **295**

12.2 Matrices as Operators **300**
An Introduction to the Determinant **300**
The Geometry of the Determinant **305**
Matrix Inversion **306**
12.3 Eigenvectors and Eigenvalues **314**
Change of Basis **316**
12.4 Further Investigation of Inner Products **319**
The Kronecker Delta Function as an Inner Product **322**
12.5 Hermitian Operators **322**
Why We Can't Measure with Complex Numbers **322**
Hermitian Operators Have Real Eigenvalues **324**
12.6 Unitary operators **326**
12.7 The Direct Sum and the Tensor Product **327**
The Direct Sum **327**
The Tensor Product **329**
12.8 Hilbert Space **333**
Metrics, Cauchy Sequences and Completeness **333**
An Axiomatic Definition of the Inner Product **337**
The Definition of Hilbert Space **338**
12.9 The Qubit as a Hilbert Space **339**

13 ***Mathematical Tools for Quantum Computing III*** **343**
13.1 Boolean Functions **343**
13.2 Logarithms and Exponentials **344**
13.3 Euler's Formula **346**

14 ***Table of Quantum Operators and Core Circuits*** **349**

Works Cited **353**