

Contents

Chapter 1. Introduction	1
Chapter 2. Contraction and Convexity via Hamiltonian Estimates: an Heuristic Argument	13
Part I. Nonlinear Diffusion Equations and Their Linearization in Dirichlet Spaces	21
Chapter 3. Dirichlet Forms, Homogeneous Spaces and Nonlinear Diffusion	23
3.1. Dirichlet forms	23
3.2. Completion of quotient spaces w.r.t. a seminorm	25
3.3. Nonlinear diffusion	27
Chapter 4. Backward and Forward Linearizations of Nonlinear Diffusion	35
Part II. Continuity Equation and Curvature Conditions in Metric Measure Spaces	43
Chapter 5. Preliminaries	45
5.1. Absolutely continuous curves, Lipschitz functions and slopes	45
5.2. The Hopf-Lax evolution formula	46
5.3. Measures, couplings, Wasserstein distance	46
5.4. W_p -absolutely continuous curves and dynamic plans	47
5.5. Metric measure spaces and the Cheeger energy	48
5.6. Entropy estimates of the quadratic moment and of the Fisher information along nonlinear diffusion equations	49
5.7. Weighted Γ -calculus	54
Chapter 6. Absolutely Continuous Curves in Wasserstein Spaces and Continuity Inequalities in a Metric Setting	59
Chapter 7. Weighted Energy Functionals along Absolutely Continuous Curves	63
Chapter 8. Dynamic Kantorovich Potentials, Continuity Equation and Dual Weighted Cheeger Energies	67
Chapter 9. The $\text{RCD}^*(K, N)$ Condition and Its Characterizations through Weighted Convexity and Evolution Variational Inequalities	69
9.1. Green functions on intervals	69
9.2. Entropies and their regularizations	74
9.3. The $\text{CD}^*(K, N)$ condition and its characterization via weighted action convexity	75

9.4.	$\text{RCD}(K, \infty)$ spaces and a criterium for $\text{CD}^*(K, N)$ via EVI	86
Part III. Bakry-Émery Condition and Nonlinear Diffusion		91
Chapter 10.	The Bakry-Émery Condition	93
10.1.	The Bakry-Émery condition for local Dirichlet forms and interpolation estimates	93
10.2.	Local and “nonlinear” characterization of the metric $\text{BE}(K, N)$ condition in locally compact spaces	96
Chapter 11.	Nonlinear Diffusion Equations and Action Estimates	99
Chapter 12.	The Equivalence Between $\text{BE}(K, N)$ and $\text{RCD}^*(K, N)$	103
12.1.	Regular curves and regularized entropies	103
12.2.	$\text{BE}(K, N)$ yields EVI for regular entropy functionals in $\text{DC}(N)$	106
12.3.	$\text{RCD}^*(K, N)$ implies $\text{BE}(K, N)$	113
Bibliography		119