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### General Introduction

#### PART I. HOMOLOGY THEORY OF POLYHEDRA

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## PART II. GENERAL HOMOLOGY THEORY

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