

Contents

<i>Preface</i>	<i>page x</i>
1 Review of mathematical notions used in the analysis of transport problems in densely-packed composite materials	1
1.1 Graphs	1
1.2 Functional spaces and weak solutions of partial differential equations	3
1.3 Duality of functional spaces and functionals	9
1.4 Differentiation in functional spaces	12
1.5 Introduction to elliptic function theory	13
1.6 Kirszbraun's theorem	18
2 Background and motivation for the introduction of network models	20
2.1 Examples of real-world problems leading to discrete network models	20
2.2 Examples of network models	22
2.3 Rigorous mathematical approaches	27
2.4 When does network modeling work?	28
2.5 History of the mathematical investigation of overall properties of high-contrast materials and arrays of bodies	35
2.6 Berryman–Borcea–Papanicolaou analysis of the Kozlov model	42
2.7 Numerical analysis of the Maxwell–Keller model	44
2.8 Percolation in disordered systems	49
2.9 Summary	50

3	Network approximation for boundary-value problems with discontinuous coefficients and a finite number of inclusions	51
3.1	Variational principles and duality. Two-sided bounds	52
3.2	Composite material with homogeneous matrix	57
3.3	Trial functions and the accuracy of two-sided bounds. Construction of trial functions for high-contrast densely-packed composite materials	63
3.4	Construction of a heuristic network model. Two-dimensional transport problem for a high-contrast composite material filled with densely packed particles	65
3.5	Asymptotically matching bounds	69
3.6	Proof of the network approximation theorem	71
3.7	Close-packing systems of bodies	88
3.8	Finish of the proof of the network approximation theorem	90
3.9	The pseudo-disk method and Robin boundary conditions	98
4	Numerics for percolation and polydispersity via network models	100
4.1	Computation of flux between two closely spaced disks of different radii using the Keller method	100
4.2	Concept of neighbors using characteristic distances	102
4.3	Numerical implementation of the discrete network approximation and fluxes in the network	104
4.4	Property of the self-similarity problem (3.2.4)–(3.2.7)	105
4.5	Numerical simulations for monodispersed composite materials. The percolation phenomenon	106
4.6	Polydispersed densely-packed composite materials	110
5	The network approximation theorem for an infinite number of bodies	116
5.1	Formulation of the mathematical model	116
5.2	Triangle–neck partition and discrete network	119
5.3	Perturbed network models	129
5.4	δ - N connectedness and δ -subgraphs	129
5.5	Properties of the discrete network	131
5.6	Variational error estimates	135
5.7	The refined lower-sided bound	136
5.8	The refined upper-sided bound	138
5.9	Construction of trial function for the upper-sided bound	138
5.10	The network approximation theorem with an error estimate independent of the total number of particles	145
5.11	Estimation of the constant in the network approximation theorem	147
5.12	A posteriori numerical error	151

6	Network method for nonlinear composites	155
6.1	Formulation of the mathematical model	156
6.2	A two-step construction of the network	157
6.3	Proofs for the domain partitioning step	163
6.4	Proofs for the asymptotic step	174
7	Network approximation for potentials of bodies	180
7.1	Formulation of the problem of approximation of potentials of bodies	180
7.2	Network approximation theorem for potentials	182
8	Application of the method of complex variables	191
8.1	\mathbb{R} -linear problem and functional equations	191
8.2	Doubly-periodic problems	204
8.3	Optimal design problem for monodispersed composites	213
8.4	Random polydispersed composite	217
	<i>References</i>	228
	<i>Index</i>	242