

# CONTENTS

Preface . . . . .	vii
Acknowledgements . . . . .	ix

## CHAPTER I

### STATIC MAXIMAL FLOW

Introduction . . . . .	1
1. Networks . . . . .	2
2. Flows in networks . . . . .	4
3. Notation . . . . .	9
4. Cuts . . . . .	10
5. Maximal flow . . . . .	11
6. Disconnecting sets and cuts . . . . .	14
7. Multiple sources and sinks . . . . .	15
8. The labeling method for solving maximal flow problems . . . . .	17
9. Lower bounds on arc flows . . . . .	22
10. Flows in undirected and mixed networks . . . . .	23
11. Node capacities and other extensions . . . . .	23
12. Linear programming and duality principles . . . . .	26
13. Maximal flow value as a function of two arc capacities . . . . .	30
References . . . . .	35

## CHAPTER II

### FEASIBILITY THEOREMS AND COMBINATORIAL APPLICATIONS

Introduction . . . . .	36
1. A supply-demand theorem . . . . .	36
2. A symmetric supply-demand theorem . . . . .	42
3. Circulation theorem . . . . .	50
4. The König-Egerváry and Menger graph theorems . . . . .	53
5. Construction of a maximal independent set of admissible cells . . . . .	55
6. A bottleneck assignment problem . . . . .	57
7. Unicursal graphs . . . . .	59
8. Dilworth's chain decomposition theorem for partially ordered sets . . . . .	61
9. Minimal number of individuals to meet a fixed schedule of tasks . . . . .	64
10. Set representatives . . . . .	67

## CONTENTS

11. The subgraph problem for directed graphs . . . . .	75
12. Matrices composed of 0's and 1's . . . . .	79
References . . . . .	91

### CHAPTER III

#### MINIMAL COST FLOW PROBLEMS

Introduction . . . . .	93
1. The Hitchcock problem . . . . .	95
2. The optimal assignment problem . . . . .	111
3. The general minimal cost flow problem . . . . .	113
4. Equivalence of Hitchcock and minimal cost flow problems . . . . .	127
5. A shortest chain algorithm . . . . .	130
6. The minimal cost supply-demand problem: non-negative directed cycle costs . . . . .	134
7. The warehousing problem . . . . .	137
8. The caterer problem . . . . .	140
9. Maximal dynamic flow . . . . .	142
10. Project cost curves . . . . .	151
11. Constructing minimal cost circulations . . . . .	162
References . . . . .	169

### CHAPTER IV

#### MULTI-TERMINAL MAXIMAL FLOWS

Introduction . . . . .	173
1. Forests, trees, and spanning subtrees . . . . .	173
2. Realization conditions . . . . .	176
3. Equivalent networks . . . . .	177
4. Network synthesis . . . . .	187
References . . . . .	191