

## CONTENTS

<b>1</b>	<b>Decision Theory and Bayesian Inference</b>	<b>1</b>
1.1	Introduction	1
1.2	The Baseline Decision Problem	1
1.3	The Moral Expectation Theorem	4
1.4	The Interpretation of Probabilities	5
1.5	Factorizations of II: Bayes' Theorem	8
1.6	Extensive Form Analysis	10
1.7	Normal or Strategic Form Analysis	12
1.8	Statistical Inference and Scientific Reporting	12
1.9	Estimation	16
1.10	Hypothesis Testing	21
1.10.1	Introduction	21
1.10.2	Classical Hypothesis Testing	22
1.10.3	Bayesian Hypothesis Testing	27
1.10.4	An Example	31
<b>2</b>	<b>Bayesian Statistics and Linear Regression</b>	<b>35</b>
2.1	Introduction	35
2.2	The Likelihood Principle	35
2.2.1	Definition	35
2.2.2	Nuisance Parameters	37
2.2.3	Stopping Rules	38
2.2.4	Identification	40
2.3	Density and Likelihood Kernels	43
2.4	Sufficient Statistics	46
2.4.1	Definition	46
2.4.2	The Exponential Family	46
2.5	Natural Conjugate Inference	48
2.5.1	General Principle	48
2.5.2	Inference in the Multivariate Normal Process	49
2.6	Reductions of Models	52
2.6.1	Reduction by Conditioning and Exogeneity	52
2.6.2	Conditioning and the Regression Model	55
2.7	Inference in the Linear Regression Model	56
2.7.1	Model and Likelihood Function	56
2.7.2	Natural Conjugate Prior Density	57
2.7.3	Posterior Densities	58
2.7.4	Predictive Densities	61
2.7.5	Tests of Linear Restrictions	62

<b>3 Methods of Numerical Integration</b>	65
3.1 Introduction	65
3.2 General Principle for Partially Linear Models	67
3.3 Deterministic Integration Methods	68
3.3.1 Simpson's Rules	69
3.3.2 Other Rules	71
3.4 Monte Carlo Methods	74
3.4.1 Direct Sampling	74
3.4.2 Importance Sampling	76
3.4.3 Markov Chain Methods	83
3.5 Conclusion	93
<b>4 Prior Densities for the Regression Model</b>	94
4.1 Introduction	94
4.2 The Elicitation of a Prior Density	94
4.2.1 Distributions Adjusted on Historical Data	95
4.2.2 Subjective Prior Information: a Discussion	97
4.2.3 The Interval Betting Method for Regression Parameters	99
4.2.4 The Predictive Method	104
4.2.5 Simplifications for Assigning Prior Covariances	106
4.3 The Quantification of Ignorance	107
4.3.1 Ancient Justifications for Ignorance Priors	108
4.3.2 Modern Justifications for Ignorance Priors	108
4.3.3 Stable Inference	109
4.3.4 Jeffreys' Invariance Principle	110
4.3.5 Non-informative Limit of a Natural Conjugate Prior	113
4.3.6 The Reference Prior	115
4.4 Restrictive Properties of the <i>NIG</i> Prior	116
4.4.1 Diffuse Prior on $\sigma^2$ and Informative Prior on $\beta$	117
4.4.2 Conflicting Information	118
4.5 Student Prior and Poly- <i>t</i> Densities	118
4.5.1 Pooling Two Independent Samples	119
4.5.2 Student Prior	122
4.5.3 A Wage Equation for Belgium	123
4.6 Special Topics	124
4.6.1 Exact Restrictions	125
4.6.2 Exchangeable Priors	126
<b>5 Dynamic Regression Models</b>	129
5.1 Introduction	129
5.2 Statistical Issues Specific to Dynamic Models	129
5.2.1 Reductions: Exogeneity and Causality	130

5.2.2 Reduction of a VAR Model to an ADL Equation	132
5.2.3 Treatment of Initial Observations	134
5.2.4 Non-stationarity	136
5.3 Inference in ADL Models	136
5.3.1 Model Specification and Posterior Analysis	136
5.3.2 Truncation to the Stationarity Region	137
5.3.3 Predictive Analysis	137
5.3.4 Inference on Long-run Multipliers	140
5.4 Models with AR Errors	143
5.4.1 Common Factor Restrictions in ADL Models	144
5.4.2 Bayesian Inference	144
5.4.3 Testing for Common Factors and Autocorrelation	146
5.5 Models with ARMA Errors	148
5.5.1 Identification Problems	148
5.5.2 The Likelihood Function	150
5.5.3 Bayesian Inference	153
5.6 Money Demand in Belgium	154
<b>6 Unit Root Inference</b>	<b>158</b>
6.1 Introduction	158
6.2 Controversies in the Literature	159
6.2.1 The Helicopter Tour	160
6.2.2 Bayesian Routes to Unit Root Testing	162
6.2.3 What Is Important?	164
6.3 Dynamic Properties of the AR(1) Model	164
6.3.1 Initial Condition	164
6.3.2 Introducing a Constant and a Trend	166
6.3.3 Trend and Cycle Decomposition	168
6.4 Pathologies in the Likelihood Functions	169
6.4.1 Definitions	169
6.4.2 The Simple AR(1) Model	169
6.4.3 The Non-linear AR(1) Model with Constant	170
6.4.4 The Linear AR(1) Model with Constant	173
6.4.5 Summary	174
6.5 The Exact Role of Jeffreys' Prior	174
6.5.1 Jeffreys' Prior Without Deterministic Terms	175
6.5.2 Choosing a Prior for the Simple AR(1) Model	178
6.5.3 Jeffreys' prior with Deterministic Terms	179
6.5.4 Playing with Singularities	180
6.5.5 Bayesian Unit Root Testing	182
6.5.6 Can We Test for a Unit Root Using a Linear Model?	184

6.6	Analysing the Extended Nelson–Plosser Data	185
6.6.1	The AR( $p$ ) Model with a Deterministic Trend	185
6.6.2	The Empirical Results	188
6.7	Conclusion	192
6.8	Appendix: Jeffreys' Prior with the Exact Likelihood	193
<b>7</b>	<b>Heteroscedasticity and ARCH</b>	197
7.1	Introduction	197
7.2	Functional Heteroscedasticity	199
7.2.1	Prior Density and Likelihood Function	199
7.2.2	Posterior Analysis	201
7.2.3	A Test of Homoscedasticity	202
7.2.4	Application to Electricity Consumption	202
7.3	ARCH Models	204
7.3.1	Introduction	204
7.3.2	Properties of ARCH Processes	205
7.3.3	Likelihood Function and Posterior Density	208
7.3.4	Predictive Densities	209
7.3.5	Application to the USD/DM Exchange Rate	211
7.3.6	Regression Models with ARCH Errors	211
7.4	GARCH Models	215
7.4.1	Properties of GARCH Processes	216
7.4.2	Extensions of GARCH Processes	217
7.4.3	Inference in GARCH Processes	219
7.4.4	Application to the USD/DM Exchange Rate	220
7.5	Stationarity and Persistence	221
7.5.1	Stationarity	221
7.5.2	Measures of Persistence	223
7.5.3	Application to the USD/DM Exchange Rate	224
7.6	Bayesian Heteroscedasticity Diagnostic	225
7.6.1	Properties of Bayesian Residuals	226
7.6.2	A Diagnostic Procedure	227
7.6.3	Applications to Electricity and Exchange Rate Data Sets	229
7.7	Conclusion	229
<b>8</b>	<b>Non-Linear Time Series Models</b>	231
8.1	Introduction	231
8.2	Inference in Threshold Regression Models	232
8.2.1	A Typology of Threshold Models	232
8.2.2	Notation	234
8.2.3	Posterior Analysis in the Homoscedastic Case	235
8.2.4	Posterior Analysis for the Heteroscedastic Case	236
8.2.5	Predictive Density for the SETAR Model	237
8.3	Pathological Aspects of Threshold Models	238

8.3.1	The Nature of the Threshold	239
8.3.2	Identification in Abrupt Transition Models	239
8.3.3	Identification in Smooth Transition Models	241
8.4	Testing for Linearity and Model Selection	244
8.4.1	Model Selection	244
8.4.2	A Linearity Test Based on the Posterior Density	245
8.4.3	A Numerical Example	247
8.5	Empirical Applications	247
8.5.1	A Consumption Function for France	248
8.5.2	United States Business Cycle Asymmetries	253
8.6	Disequilibrium Models	256
8.6.1	Maximum Likelihood Estimation	257
8.6.2	The Structure of the Posterior Density	258
8.6.3	Elicitation of Prior Information on $\beta$	260
8.6.4	Numerical Evaluation of the Posterior Density	261
8.6.5	Endogenous Prices and Other Regime Indicators	262
8.7	Conclusion	263
9	<b>Systems of Equations</b>	265
9.1	Introduction	265
9.2	VAR Models	265
9.2.1	Unrestricted VAR Models and Multivariate Regression	265
9.2.2	Restricted VAR Models and SURE Models	267
9.2.3	The Minnesota Prior for VAR Models	269
9.3	Cointegration and VAR Models	272
9.3.1	Model Formulation	272
9.3.2	Identification Issues	273
9.3.3	Likelihood Function and Prior Density	274
9.3.4	Posterior Results	275
9.3.5	Examples	278
9.3.6	Selecting the Cointegration Rank	283
9.4	Simultaneous Equation Models	285
9.4.1	Limited Information Analysis	285
9.4.2	Full Information Analysis	287
<b>A</b>	<b>Probability Distributions</b>	289
A.1	Univariate Distributions	289
A.1.1	The Uniform Distribution	289
A.1.2	The Gamma, Chi-squared, and Beta Distributions	290
A.1.3	The Univariate Normal Distribution	293
A.1.4	Distributions Related to the Univariate Normal Distribution	294

A.2	Multivariate Distributions	297
A.2.1	Preliminary: Choleski Decomposition	297
A.2.2	The Multivariate Normal Distribution	298
A.2.3	The Matricvariate Normal Distribution	301
A.2.4	The Normal-Inverted Gamma-2 Distribution	302
A.2.5	The Multivariate Student Distribution	303
A.2.6	The Inverted Wishart Distribution	305
A.2.7	The Matricvariate Student Distribution	307
A.2.8	Poly- <i>t</i> Distributions	309
B	Generating random numbers	312
B.1	General Methods for Univariate Distributions	312
B.1.1	Inverse Transform Method	313
B.1.2	Acceptance-Rejection Method	313
B.1.3	Compound or Data Augmentation Method	315
B.2	Univariate Distributions	315
B.2.1	Exponential Distribution	315
B.2.2	Gamma Distribution	316
B.2.3	Chi-squared Distribution	316
B.2.4	Inverted Gamma-2 Distribution	317
B.2.5	Beta Distribution	317
B.2.6	Normal Distribution	317
B.2.7	Student Distribution	317
B.2.8	Cauchy Distribution	318
B.3	General Methods for Multivariate Distributions	318
B.3.1	Multivariate Transformations	318
B.3.2	Factorization into Marginals and Conditionals	319
B.3.3	Markov Chains	319
B.4	Multivariate Distributions	319
B.4.1	Multivariate Normal	319
B.4.2	Multivariate Student	320
B.4.3	Matricvariate Normal	320
B.4.4	Inverted Wishart	320
B.4.5	Matricvariate Student	321
B.4.6	Poly- <i>t</i> 2-0	321
B.4.7	Poly- <i>t</i> 1-1	322
	References	323
	Subject Index	340
	Author Index	347