

# Contents

*A comment on page numbering: almost every module begins with a one-page picture that is also a math/art puzzle meant to be removed for ease of drawing. The in-class worksheet follows immediately after.*

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