
Contents

Preface	ix
Part I. Vector spaces	
Chapter 1. The basics	3
1.1. The vector space \mathbb{F}^n	3
1.2. Linear combinations	7
1.3. Matrices and the equation $Ax = b$	11
1.4. The basic counting theorem	16
1.5. Matrices and linear transformations	21
1.6. Exercises	25
Chapter 2. Systems of linear equations	29
2.1. The geometry of linear systems	29
2.2. Solving systems of equations—setting up	36
2.3. Solving linear systems—echelon forms	40
2.4. Solving systems of equations—the reduction process	44
2.5. Drawing some consequences	50
2.6. Exercises	54
Chapter 3. Vector spaces	57
3.1. The notion of a vector space	57
3.2. Linear combinations	62
3.3. Bases and dimension	70
3.4. Subspaces	79
3.5. Affine subspaces and quotient vector spaces	91
3.6. Exercises	95

Chapter 4. Linear transformations	103
4.1. Linear transformations I	103
4.2. Matrix algebra	107
4.3. Linear transformations II	112
4.4. Matrix inversion	122
4.5. Looking back at calculus	128
4.6. Exercises	132
Chapter 5. More on vector spaces and linear transformations	139
5.1. Subspaces and linear transformations	139
5.2. Dimension counting and applications	142
5.3. Bases and coordinates: vectors	148
5.4. Bases and matrices: linear transformations	158
5.5. The dual of a vector space	167
5.6. The dual of a linear transformation	171
5.7. Exercises	177
Chapter 6. The determinant	195
6.1. Volume functions	195
6.2. Existence, uniqueness, and properties of the determinant	202
6.3. A formula for the determinant	206
6.4. Practical evaluation of determinants	211
6.5. The classical adjoint and Cramer's rule	213
6.6. Jacobians	214
6.7. Exercises	216
Chapter 7. The structure of a linear transformation	221
7.1. Eigenvalues, eigenvectors, and generalized eigenvectors	222
7.2. Polynomials in \mathcal{T}	226
7.3. Application to differential equations	234
7.4. Diagonalizable linear transformations	239
7.5. Structural results	246
7.6. Exercises	253
Chapter 8. Jordan canonical form	259
8.1. Chains, Jordan blocks, and the (labelled) eigenstructure picture of \mathcal{T}	260
8.2. Proof that \mathcal{T} has a Jordan canonical form	263
8.3. An algorithm for Jordan canonical form and a Jordan basis	268
8.4. Application to systems of first-order differential equations	275
8.5. Further results	280
8.6. Exercises	283

Part II. Vector spaces with additional structure

Chapter 9. Forms on vector spaces	291
9.1. Forms in general	291
9.2. Usual types of forms	298
9.3. Classifying forms I	301
9.4. Classifying forms II	303
9.5. The adjoint of a linear transformation	310
9.6. Applications to algebra and calculus	317
9.7. Exercises	319
Chapter 10. Inner product spaces	325
10.1. Definition, examples, and basic properties	325
10.2. Subspaces, complements, and bases	331
10.3. Two applications: symmetric and Hermitian forms, and the singular value decomposition	339
10.4. Adjoints, normal linear transformations, and the spectral theorem	350
10.5. Exercises	359
Appendix A. Fields	367
A.1. The notion of a field	367
A.2. Fields as vector spaces	368
Appendix B. Polynomials	371
B.1. Statement of results	371
B.2. Proof of results	373
Appendix C. Normed vector spaces and questions of analysis	377
C.1. Spaces of sequences	377
C.2. Spaces of functions	379
Appendix D. A guide to further reading	383
Index	385