

Contents

1	Introduction	1
1.1	An Overview	1
1.2	Notes on the Bibliography	8
2	The Matrix Schrödinger Equation and the Characterization of the Scattering Data	19
2.1	Outline of the Chapter	19
2.2	The Matrix Schrödinger Equation on the Half Line	22
2.3	Star Graphs	26
2.4	The Schrödinger Equation on the Full Line	28
2.5	The Faddeev Class and the Marchenko Class	31
2.6	A First Characterization of the Scattering Data	38
2.7	Alternate Characterizations of the Scattering Data	40
2.8	Another Characterization of the Scattering Data	45
3	Direct Scattering I	49
3.1	Outline of the Solution to the Direct Problem	49
3.2	Special Solutions to the Schrödinger Equation	51
3.3	The Hamiltonian	90
3.4	Equivalence of the Formulations of the Boundary Condition	94
3.5	The Quadratic Form of the Hamiltonian	103
3.6	Transformations of the Jost and Scattering Matrices	109
3.7	The Jost and Scattering Matrices with Zero Potential	112
3.8	Low-Energy Analysis with Potentials in the Faddeev Class	114
3.9	Low-Energy Analysis with Potentials of Finite Second Moment	155
3.10	High-Energy Analysis	191
3.11	Bound States	199
3.12	Levinson's Theorem	222
3.13	Further Properties of the Scattering Data	226
3.14	The Marchenko Integral Equation	231

3.15	The Boundary Matrices	237
3.16	The Existence and Uniqueness in the Direct Problem	240
4	Direct Scattering II	261
4.1	Basic Principles of the Scattering Theory	261
4.2	The Limiting Absorption Principle	265
4.3	The Generalized Fourier Maps for the Absolutely Continuous Subspace	276
4.4	The Wave Operators	289
4.5	The Scattering Operator and the Scattering Matrix	298
4.6	The Spectral Shift Function	300
4.7	Trace Formulas	313
4.8	The Number of Bound States	322
5	Inverse Scattering	339
5.1	Nonuniqueness Due to the Improperly Defined Scattering Matrix	340
5.2	The Solution to the Inverse Problem	346
5.3	Bounds on the Constructed Solutions	380
5.4	Relations Among the Characterization Conditions	390
5.5	The Proof of the First Characterization Theorem	400
5.6	Equivalents for Some Characterization Conditions	403
5.7	Inverse Problem Using Only the Scattering Matrix as Input	413
5.8	Characterization via Levinson's Theorem	417
5.9	Parseval's Equality	445
5.10	The Generalized Fourier Map	448
5.11	An Alternate Method to Solve the Inverse Problem	459
5.12	Characterization with Potentials of Stronger Decay	475
5.13	The Dirichlet Boundary Condition	478
6	Some Explicit Examples	485
6.1	Illustration of the Theory with Explicit Examples	485
6.2	Some Methods Yielding Explicit Examples	486
6.3	Explicit Examples in the Characterization of the Scattering Data ..	496
6.4	Explicit Examples of Particular Solutions	536
Appendix A	Mathematical Preliminaries	545
A.1	Vectors, Matrices, and Functions	545
A.2	Banach and Hilbert Spaces	548
A.3	Inequalities	553
A.4	Mollifiers	555
A.5	Equicontinuity	556
A.6	Distributions	556
A.7	Absolute Continuity	559
A.8	Sobolev Spaces	561
A.9	The Fourier Transform	562

A.10	Hardy Spaces	564
A.11	Other Banach Spaces	566
A.12	Linear Operators Between Banach and Hilbert Spaces	567
A.13	Operators Between Finite Dimensional Hilbert Spaces	572
A.14	Self-adjoint Operators and Symmetric Quadratic Forms	573
A.15	Trace-Class and Hilbert–Schmidt Operators	576
A.16	Resolvent and Spectrum	580
A.17	The Spectral Theorem	580
A.18	The Spectral Shift Function	587
A.19	Deficiency Indices	589
A.20	Self-adjoint Extensions of Matrix Schrödinger Operators	590
A.21	Hermitian Symplectic Geometry	592
A.22	Integral Operators	596
A.23	The Gronwall Inequalities	602
A.24	Miscellaneous Results	604
A.25	Notes	605
References		607
Index		619
List of Symbols		623