Game Theory

Second Edition

MICHAEL MASCHLER
EILON SOLAN
SHMUEL ZAMIR

Translated from Hebrew by Ziv Hellman

English Editor Mike Borns

CAMBRIDGE
UNIVERSITY PRESS

CONTENTS

Acknowledgments xiv
Notations xv
Introduction xxiii

1 The game of chess 1

1.1 Schematic description of the game 1
1.2 Analysis and results 2
1.3 Remarks 7
1.4 Exercises 7

2 Utility theory 9

2.1 Preference relations and their representation 9
2.2 Preference relations over uncertain outcomes: the model 12
2.3 The axioms of utility theory 14
2.4 The characterization theorem for utility functions 19
2.5 Utility functions and affine transformations 22
2.6 Infinite outcome set 23
2.7 Attitude towards risk 23
2.8 Subjective probability 26
2.9 Discussion 27
2.10 Remarks 31
2.11 Exercises 31

3 Extensive-form games 39

3.1 An example 40
3.2 Graphs and trees 41
3.3 Game trees 42
3.4 Chomp: David Gale's game 47
3.5 Games with chance moves 49
3.6 Games with imperfect information 51
3.7 Exercises 57

4 Strategic-form games 75

4.1 Examples and definition of strategic-form games 76
4.2 The relationship between the extensive form and the strategic form 81
4.3 Strategic-form games: solution concepts 84
4.4 Notation 85
4.5 Domination 85
4.6 Second-price auctions 91
4.7 The order of elimination of dominated strategies 95
4.8 Stability: Nash equilibrium 95
4.9 Properties of the Nash equilibrium 100
4.10 Security: the maxmin concept 102
4.11 The effect of elimination of dominated strategies 105
4.12 Two-player zero-sum games 110
4.13 Games with perfect information 117
4.14 Games on the unit square 121
4.15 Remarks 127
4.16 Exercises 127

5 Mixed strategies 143

5.1 The mixed extension of a strategic-form game 144
5.2 Computing equilibria in mixed strategies 151
5.3 The proof of Nash's Theorem 165
5.4 Generalizing Nash's Theorem 169
5.5 Utility theory and mixed strategies 171
5.6 The maxmin and the minmax in n-player games 175
5.7 Imperfect information: the value of information 179
5.8 Rationalizability 185
5.9 Evolutionarily stable strategies 190
5.10 The dependence of Nash equilibria on the payoffs of the game 197
5.11 Remarks 202
5.12 Exercises 203

6 Behavior strategies and Kuhn's Theorem 230

6.1 Behavior strategies 232
6.2 Kuhn's Theorem 236
6.3 Equilibria in behavior strategies 246
6.4 Kuhn's Theorem for infinite games 249
6.5 Remarks 254
6.6 Exercises 255

7 Equilibrium refinements 262

7.1 Subgame perfect equilibrium 263
7.2 Rationality, backward induction, and forward induction 271
7.3 Perfect equilibrium 273
7.4 Sequential equilibrium 282
7.5 Remarks 294
7.6 Exercises 294

8 Correlated equilibria 311

8.1 Examples 312
8.2 Definition and properties of correlated equilibrium 316
8.3 Remarks 324
8.4 Exercises 324

9 Games with incomplete information and common priors 330

9.1 The Aumann model of incomplete information and the concept of knowledge 333
9.2 The Aumann model of incomplete information with beliefs 344
9.3 An infinite set of states of the world 354
9.4 The Harsanyi model of games with incomplete information 356
9.5 Incomplete information as a possible interpretation of mixed strategies 371
9.6 The common prior assumption: inconsistent beliefs 375
9.7 Remarks 377
9.8 Exercises 378

10 Games with incomplete information: the general model 396

10.1 Belief spaces 396
10.2 Belief and knowledge 401
10.3 Examples of belief spaces 404
10.4 Belief subspaces 410
10.5 Games with incomplete information 416
10.6 The concept of consistency 424
10.7 Remarks 432
10.8 Exercises 432

11 The universal belief space 449

11.1 Belief hierarchies 451
11.2 Types 459
11.3 Definition of the universal belief space 462

11.4 Remarks 464
11.5 Exercises 465

12 Auctions 469

12.1 Notation 472
12.2 Common auction methods 472
12.3 Definition of a sealed-bid auction with private values 473
12.4 Equilibrium 476
12.5 The symmetric model with independent private values 479
12.6 The Envelope Theorem 492
12.7 Risk aversion 496
12.8 Mechanism design 500
12.9 Individually rational mechanisms 508
12.10 Finding the optimal mechanism 509
12.11 Remarks 516
12.12 Exercises 516

13 Repeated games 527

13.1 The model 528
13.2 Examples 529
13.3 The T-stage repeated game 532
13.4 Characterization of the set of equilibrium payoffs of the T-stage repeated game 538
13.5 Infinitely repeated games 545
13.6 The discounted game 550
13.7 Subgame perfectness 554
13.8 Uniform equilibrium 556
13.9 Discussion 564
13.10 Remarks 565
13.11 Exercises 565

14 Repeated games with vector payoffs 578

14.1 Notation 579
14.2 The model 581
14.3 Examples 582
14.4 Connections between approachable and excludable sets 583
14.5 A geometric condition for the approachability of a set 584
14.6 Characterizations of convex approachable sets 594
14.7 Application 1: Repeated games with incomplete information 598
14.8 Application 2: Challenge the expert 608
14.9 Discussion 614
14.10 Remarks 615
14.11 Exercises 616

15 Stochastic games 631

15.1 The model 632
15.2 The T-stage stochastic game 636
15.3 The infinite discounted game 643
15.4 Remarks 661
15.5 Exercises 662

16 Bargaining games 673

16.1 Notation 676
16.2 The model 676
16.3 Properties of the Nash solution 677
16.4 Existence and uniqueness of the Nash solution 681
16.5 Another characterization of the Nash solution 686
16.6 The minimality of the properties of the Nash solution 690
16.7 Critiques of the properties of the Nash solution 691
16.8 Monotonicity properties 693
16.9 Bargaining games with more than two players 701
16.10 Remarks 703
16.11 Exercises 704

17 Coalitional games with transferable utility 709

17.1 Examples 711
17.2 Strategic equivalence 718
17.3 A game as a vector in a Euclidean space 720
17.4 Special families of games 721
17.5 Solution concepts 722
17.6 Geometric representation of the set of imputations 725
17.7 Remarks 727
17.8 Exercises 727

18 The core 735

18.1 Definition of the core 736
18.2 Balanced collections of coalitions 739
18.3 The Bondareva–Shapley Theorem 744
18.4 Market games 750
18.5 Additive games 760
18.6 The consistency property of the core 762
18.7 Convex games 765
18.8 Spanning tree games 768
18.9 Flow games 772
18.10 The core for general coalitional structures 780

18.11 Remarks 782
18.12 Exercises 783

19 The Shapley value 796

19.1 The Shapley properties 797
19.2 Solutions satisfying some of the Shapley properties 800
19.3 The definition and characterization of the Shapley value 802
19.4 Examples 806
19.5 An alternative characterization of the Shapley value 808
19.6 Application: the Shapley–Shubik power index 811
19.7 Convex games 815
19.8 The consistency of the Shapley value 816
19.9 Remarks 822
19.10 Exercises 822

20 The bargaining set 830

20.1 Definition of the bargaining set 832
20.2 The bargaining set in two-player games 836
20.3 The bargaining set in three-player games 836
20.4 The bargaining set in convex games 842
20.5 Discussion 845
20.6 Remarks 846
20.7 Exercises 846

21 The nucleolus 849

21.1 Definition of the nucleolus 850
21.2 Nonemptiness and uniqueness of the nucleolus 853
21.3 Properties of the nucleolus 857
21.4 Computing the nucleolus 862
21.5 Characterizing the prenucleolus 863
21.6 The consistency of the nucleolus 870
21.7 Weighted majority games 873
21.8 The bankruptcy problem 879
21.9 Discussion 890
21.10 Remarks 891
21.11 Exercises 892

22 Social choice 901

22.1 Social welfare functions 904
22.2 Social choice functions 912

22.3 Non-manipulability 918
22.4 Discussion 921
22.5 Remarks 921
22.6 Exercises 922

23 Stable matching 931

23.1 The model 933
23.2 Existence of stable matching: the men's courtship algorithm 935
23.3 The women's courtship algorithm 937
23.4 Comparing matchings 939
23.5 Extensions 945
23.6 Remarks 951
23.7 Exercises 952

24 Appendices 962

24.1 Fixed point theorems 962
24.2 The Separating Hyperplane Theorem 989
24.3 Linear programming 991
24.4 Remarks 995
24.5 Exercises 996

References 1004
Index 1014