

Contents

1	Introduction	1
Part I Motivic Coarse Spaces and Spectra		
2	Bornological Coarse Spaces	13
2.1	Basic Definitions	13
2.2	Examples	15
2.3	Categorical Properties of BornCoarse	17
3	Motivic Coarse Spaces	21
3.1	Descent	22
3.2	Coarse Equivalences	26
3.3	Flasque Spaces	28
3.4	u -Continuity and Motivic Coarse Spaces	31
3.5	Coarse Excision and Further Properties	33
4	Motivic Coarse Spectra	35
4.1	Stabilization	35
4.2	Further Properties of Yo^s	40
4.3	Homotopy Invariance	44
4.4	Axioms for a Coarse Homology Theory	48
5	Merging Coarse and Uniform Structures	53
5.1	The Hybrid Structure	53
5.2	Decomposition Theorem	58
5.2.1	Uniform Decompositions and Statement of the Theorem	58
5.2.2	Proof of the Decomposition Theorem	60
5.2.3	Excisiveness of the Cone-at-Infinity	65
5.3	Homotopy Theorem	65
5.3.1	Statement of the Theorem	66
5.3.2	Proof of the Homotopy Theorem	66
5.3.3	Uniform Homotopies and the Cone Functors	70
5.4	Flasque Hybrid Spaces	72

5.5	Decomposition of Simplicial Complexes	77
5.5.1	Metrics on Simplicial Complexes	77
5.5.2	Decomposing Simplicial Complexes	79
5.6	Flasqueness of the Coarsening Space	82
5.6.1	Construction of the Coarsening Space	82
5.6.2	Flasqueness for the \mathcal{C}_0 -Structure	85
5.6.3	Flasqueness for the Hybrid Structure	87
5.7	The Motivic Coarse Spectra of Simplicial Complexes and Coarsening Spaces	91

Part II Coarse and Locally Finite Homology Theories

6	First Examples and Comparison of Coarse Homology Theories	95
6.1	Forcing u -Continuity	95
6.2	Additivity and Coproducts	98
6.2.1	Additivity	98
6.2.2	Coproducts	99
6.3	Coarse Ordinary Homology	101
6.4	Coarsification of Stable Homotopy	106
6.4.1	Rips Complexes and a Coarsification of Stable Homotopy	108
6.4.2	Proof of Theorem 6.32	112
6.4.3	Further Properties of the Functor Q and Generalizations ...	115
6.5	Comparison of Coarse Homology Theories	117
7	Locally Finite Homology Theories and Coarsification	119
7.1	Locally Finite Homology Theories	119
7.1.1	Topological Bornological Spaces	120
7.1.2	Definition of Locally Finite Homology Theories	122
7.1.3	Additivity	129
7.1.4	Construction of Locally Finite Homology Theories	132
7.1.5	Classification of Locally Finite Homology Theories	135
7.2	Coarsification of Locally Finite Theories	138
7.3	Analytic Locally Finite K -Homology	141
7.3.1	Extending Functors from Locally Compact Spaces to TopBorn	141
7.3.2	Cohomology for C^* -Algebras	145
7.3.3	Locally Finite Homology Theories from Cohomology Theories for C^* -Algebras	147
7.4	Coarsification Spaces	150
8	Coarse K-Homology	157
8.1	X -Controlled Hilbert Spaces	158
8.2	Ample X -Controlled Hilbert Spaces	161
8.3	Roe Algebras	166
8.4	K -Theory of C^* -Algebras	174

8.5	<i>C</i> [*] -Categories and Their <i>K</i> -Theory	177
8.5.1	Definition of <i>C</i> [*] -Categories	179
8.5.2	From <i>C</i> [*] -Categories to <i>C</i> [*] -Algebras and <i>K</i> -Theory	180
8.5.3	<i>K</i> -Theory Preserves Filtered Colimits	185
8.5.4	<i>K</i> -Theory Preserves Unitary Equivalences	185
8.5.5	Exactness of <i>K</i> -Theory	187
8.5.6	Additivity of <i>K</i> -Theory	190
8.6	Coarse <i>K</i> -Homology	193
8.7	Comparison with the Classical Definition	203
8.8	Additivity and Coproducts	212
8.8.1	Additivity	212
8.8.2	Coproducts	221
8.9	Dirac Operators	224
8.10	<i>K</i> -Theoretic Coarse Assembly Map	230
	References	235
	Index	239