

Contents

Chapter 1. Introduction	1
Chapter 2. The group theoretical formalism	7
2.1. Γ -graduations on smooth affine groups	7
2.2. Γ -filtrations on reductive groups	10
2.3. Relative positions of Γ -filtrations	18
2.4. Interlude on the dominance partial orders	22
Chapter 3. The Tannakian formalism	31
3.1. Γ -graduations and Γ -filtrations on quasi-coherent sheaves	32
3.2. Γ -graduations and Γ -filtrations on fiber functors	33
3.3. The subcategories of rigid objects	37
3.4. Skalar extensions	39
3.5. The regular representation	42
3.6. Relating $\text{Rep}(G)(S)$ and $\text{Rep}^\circ(G)(S)$	44
3.7. The stabilizer of a Γ -filtration, I	47
3.8. Grothendieck groups	52
3.9. The stabilizer of a Γ -filtration, II	53
3.10. Splitting filtrations	55
3.11. Consequences	61
3.12. Ranks and relative positions	69
3.13. Appendix: pure subsheaves	70
Chapter 4. The vectorial Tits building $\mathbf{F}^\Gamma(G)$	73
4.1. Combinatorial structures	73
4.2. Distances and angles	80
Chapter 5. Affine $\mathbf{F}(G)$ -buildings	87
5.1. The dominance order	87
5.2. Affine $\mathbf{F}(G)$ -spaces and buildings	89
5.3. Further axioms	97
5.4. Walls and tight buildings	101
5.5. Metric properties	105
5.6. The affine $\mathbf{F}(P/U)$ -space $\mathbf{T}_P^\infty \mathbf{X}(G)$	109
5.7. Example: $\mathbf{F}(G)$ as a tight affine $\mathbf{F}(G)$ -building	113
5.8. Example: a symmetric space	116
Chapter 6. Bruhat-Tits buildings	121
6.1. The Bruhat-Tits building of $GL(V)$	121
6.2. The Bruhat-Tits building of G	123
6.3. Functoriality for Bruhat-Tits buildings	125

6.4. A Tannakian formalism for Bruhat-Tits buildings	129
Nomenclature	139
Bibliography	145