Contents

1	Information Granules and Granular Computing		
	1.1	Information Granules and Information Granularity	2
	1.2	Frameworks of Information Granules	4
	1.3	Explainable Artificial Intelligence	6
	1.4	Image Understanding and Computer Vision	8
	1.5	Data Analytics	8
	1.6	Conclusions	10
	Refere	ences	11
2	Key Concepts of Fuzzy Sets		13
	2.1	Sets and Interval Analysis	13
	2.2	Fuzzy Sets: A Departure from the Principle	
		of Dichotomy	15
	2.3	Classes of Membership Functions	17
	2.4	Selected Descriptors of Fuzzy Sets	20
	2.5	A Representation Theorem	27
	2.6	Families of Fuzzy Sets: A Frame of Cognition	30
	2.7	Linguistic Modifiers and Linguistic Quantifiers	30
	2.8	Linguistic Approximation	31
	2.9	Linguistic Variables	32
	2.10	Conclusions	33
	Refere	ences	35
3	Interval Calculus		37
	3.1	Operations on Intervals	38
	3.2	Distance Between Intervals	40
	3.3	Mapping of Interval Arguments	42
	3.4	Relations and Relation Calculus	43

	3.5 3.6 Referen	Composition Operations	45 47 48
4	Fuzzy 4.1 4.2 4.3 4.4 4.5 4.6 Refere	Relations and Relational Computing Fuzzy Relations Properties of Fuzzy Relations Operations on Fuzzy Relations Equivalence and Similarity Relations Fuzzy Relations Conclusions ences	49 52 53 59 60 64 65
5	Fuzzy 5.1 5.2 5.3	Set Transformation and Fuzzy ArithmeticThe Extension PrincipleFuzzy Numbers and Their Arithmetic5.2.1Computing with Fuzzy Numbers:The Representation Theorem Approach5.2.2Computing with Fuzzy Numbers: The ExtensionPrinciple Approach5.2.3Addition5.2.4Multiplication5.2.5DivisionConclusions	67 69 69 73 74 75 76 76 76
6	Oper 6.1 6.2 6.3 6.4 6.5 Refer	ations on Fuzzy SetsGeneric Operations on Fuzzy SetsTriangular Norms and Triangular Co-norms as Modelsof Operations on Fuzzy SetsNegationsAggregation Operators6.4.1Ordered Weighted Average6.4.2Compensatory Operators6.4.3Averaging Operator6.4.4Sensitivity Analysis of Aggregation Operators6.4.5Extreme Sensitivity6.4.6Average SensitivityConclusions	 79 79 79 83 87 87 88 89 90 90 90 90 90 91 91 93
7	High 7.1 7.2 7.3	her Type, Higher Order Fuzzy Sets and Hybrid Fuzzy Sets Fuzzy Sets of Higher Order Rough Fuzzy Sets and Fuzzy Rough Sets Type-2 Fuzzy Sets	95 95 97 99

	7.4	Interval-Valued Fuzzy Sets	101
	7.5	Probabilistic Sets	102
	7.6	Hybrid Models of Information Granules: Probabilistic	
		and Fuzzy Set Information Granules	103
	7.7	Conclusions	106
	Referen	nces	107
8	User-O	Priented Elicitation of Membership	
0	Functio	ons	109
	8.1	Semantics of Fuzzy Sets: Some General	
	0	Observations	110
	8.2	Fuzzy Set as a Descriptor of Feasible	
	0.2	Solutions	111
	8.3	Fuzzy Set as a Descriptor of the Notion	
		of Typicality	115
	8.4	Vertical and Horizontal Schemes of Membership Function	
		Estimation	117
	8.5	Saaty's Priority Approach of Pairwise Membership Function	
		Estimation	119
	8.6	Conclusions	122
	Refere	nces	124
0	Engen	Clustering	125
9	FUZZy	From Data to Clusters: Structure Discovery	125
	9.1	The Clustering Problem: A Formal Formulation	127
	9.2	The Fuzzy C-Means Algorithm	128
	9.4	Main Parameters of the FCM Algorithm	133
	9.5	Construction of One-Dimensional Fuzzy Sets	135
	9.6	Structural View of the Clusters	136
	9.7	Internal and External Measures of Quality of Fuzzy	
	2.1	Clusters	137
	9.8	Distance Functions in the FCM Algorithm	139
	9.9	Conditional Fuzzy C-Means.	142
	9.10	Conclusions	143
	Refere	nces	145
10	The P	rinciple of Justifiable Granularity	147
10	10.1	The Main Idea	148
	10.2	Design of Fuzzy Sets	152
	10.3	General Observations: An Elevation Effect	154
	10.5	The Principle of Justified Granularity in the Presence	
	10.4	of Weighted and Inhibitory Data	154
	10.5	Adversarial Information Granules	155
	10.6	Designing Fuzzy Sets of Type-2	156
	10.7	The Accommodation of Domain Knowledge	156

	10.8	Collaborative Development of Information	1.57
	10.9	Fuzzy Clustering and the Principle of Justifiable	157
	10.10	Conclusions	150
	Referen	nces	160
11	Granu	lation-Degranulation Processes	161
	11.1	The Concept of Information Granulation and	101
		Degranulation	161
		11.1.1 Granulation	161
		11.1.2 Degranulation	163
	11.2	Degranulation Process—A One Dimensional Case	164
	11.5	Degranulation Process with the Clustering	1.44
	114	Granulation and Degranulation for Granular Data	166
	11.4	11.4.1 Degranulation	160
	11.5	Conclusions	171
	Referen	nces	172
12	Fuzzy	Models: Fundamentals and Design	175
	12.1	Fuzzy Models: A Concept	175
	~	12.1.1 Input Interface	176
		12.1.2 Output interface	177
		12.1.3 Center of Gravity	178
		12.1.4 Processing Module	180
	12.2	The Design of Fuzzy Models	181
		12.2.1 Internal Level of the Design	181
	12.2	12.2.2 External Level of the Design	182
	12.5	Categories of Fuzzy Models: An Overview	183
	12.4	Bias-Variance Trade-off	101
	12.5	Conclusions	191
	Referen	nces	193
13	Fuzzy	Rule-Based Models	107
15	13.1	The Principles of Rule-Based Computing	197
	13.2	Relational Rule-Based Models	197
		13.2.1 Representation of Rules	190
		13.2.2 Types of Rules	202
		13.2.3 Quality of Rules	202
	13.3	Functional Rule-Based Models	205
		13.3.1 Input-Output Characteristics of Rule-Based Model	207
	13.4	The Design of the Rule-Based Model	208

xii

	13.5	Some Design Alternatives of the Rule-Based Models	210
		13.5.2 Modifications to the Logal Europtians	210
	13.6	Fuzzy Controllers	212
	15.0	13.6.1 Advantages	212
	13.7	Hierarchy of Fuzzy Controllere	216
	13.8	Functional Rule-Based Model of Variable State of the RUD	216
		Controller	210
	13.9	Rule-Based Models for Highly-Dimensional Data	218
	1017	13.9.1 Design and Aggregation of One-Dimensional	218
		Rule-Based Models	218
		13.9.2 Concentration Effect	210
	13.10	Conclusions	219
	Refere	nces	225
• •		N	224
14	Fuzzy	Neurocomputing	225
	14.1	Neural Networks: Concise Prerequisites	225
	14.2	Fuzzy Sets and Neural Networks: A Synergy	227
	14.3	Main Classes of Fuzzy Neurons	227
		14.3.1 Aggregative Neurons	227
	14.4	14.3.2 Referential Neurons	230
	14.4	Logic Processor: A Generic Architecture of Logic-Based	
	145		232
	14.5	Learning of Fuzzy Neural Networks	233
	14.0	Logic Network in Modeling the Connection S Date	235
	14.7	Eugra Relational Easterization	238
	14.0	Fuzzy Relational Factorization	239
	14.9	14.0.1 Eurzy Bula Bayed Model with a Lasia	240
		Autoencoder	~ ~ ~
		14.9.2 Design with Relational Easteringtion	241
	14-10	Conclusions	244
	Referen		245
	Referen		240
15	Applic	ations: Granular Models, Granular Classifiers and Fuzzy	
	Cognit	ive Maps	249
	15.1	Granular Models	249
		15.1.1 Granular Model—A Design with the Use of the	
		Augmented Principle of Justifiable Granularity	250
		15.1.2 Determining Information Granules in the Input	
		Space Through Mapping to One-Dimensional	
		Spaces	252
		15.1.3 Development of Granular Fuzzy Models with the	
		Use of Conditional FCM	253

15.2	Granular Classifiers	256
15.3	Fuzzy Cognitive Maps	260
	15.3.1 The Design of the Map	262
15.4	Conclusions	267
Refere	nces	269
Appendix .	A: Optimization	271
Appendix	B: Final Examinations	277