Contents

Preface vii

Chapter 1. Topological surfaces 1

§1.1. Topological and smooth manifolds 1

§1.2. PL structures 12

§1.3. Planar representations of surfaces 28

§1.4. Orientability 34

§1.5. Classification of compact surfaces 41

Problems 47

References and extra material 49

Chapter 2. Algebraic topology 53

§2.1. Homotopy theory 53

§2.2. Covers 69

§2.3. Singular homology 84

§2.4. Simplicial homology 94

§2.5. De Rham cohomology 104

§2.6. Poincaré duality 120

Problems 131

References and extra material 133

Chapter 3. Riemannian geometry 137

§3.1. Riemannian metrics. Curvature. Geodesics 137

§3.2. Riemannian surfaces and the Gauss-Bonnet theorem 160

§3.3. Isotropic, symmetric, and homogeneous manifolds 181

Problems 192

References and extra material 194

Chapter 4. Constant curvature 197
§4.1. Positive constant curvature 197
§4.2. Vanishing constant curvature 217
§4.3. Negative constant curvature 232
§4.4. Classical geometries 258
Problems 260
References and extra material 262

Chapter 5. Complex geometry 265
§5.1. Complex manifolds 265
§5.2. Kähler manifolds 284
§5.3. Complex curves 300
§5.4. Classification of complex curves 315
§5.5. Elliptic curves 323
Problems 337
References and extra material 340

Chapter 6. Global analysis 343
§6.1. Conformal structures 343
§6.2. Hodge theory 350
§6.3. Metrics of constant curvature 362
§6.4. The curvature flow 374
Problems 386
References and extra material 388

Subject Index 391

Symbols 403