

# Contents

<b>1</b>	<b>Introduction</b> . . . . .	1
1.1	What Is Nonlinear Model Predictive Control? . . . . .	1
1.2	Where Did NMPC Come From? . . . . .	3
1.3	How Is This Book Organized? . . . . .	5
1.4	What Is Not Covered in This Book? . . . . .	9
	References . . . . .	10
<b>2</b>	<b>Discrete Time and Sampled Data Systems</b> . . . . .	13
2.1	Discrete Time Systems . . . . .	13
2.2	Sampled Data Systems . . . . .	16
2.3	Stability of Discrete Time Systems . . . . .	29
2.4	Stability of Sampled Data Systems . . . . .	37
2.5	Notes and Extensions . . . . .	40
	Problems . . . . .	41
	References . . . . .	43
<b>3</b>	<b>Nonlinear Model Predictive Control</b> . . . . .	45
3.1	The Basic NMPC Algorithm . . . . .	45
3.2	Constraints . . . . .	48
3.3	Variants of the Basic NMPC Algorithms . . . . .	52
3.4	The Dynamic Programming Principle . . . . .	58
3.5	Notes and Extensions . . . . .	64
	Problems . . . . .	67
	References . . . . .	68
<b>4</b>	<b>Infinite Horizon Optimal Control</b> . . . . .	71
4.1	Definition and Well Posedness of the Problem . . . . .	71
4.2	The Dynamic Programming Principle . . . . .	74
4.3	Relaxed Dynamic Programming . . . . .	80
4.4	Notes and Extensions . . . . .	86
	Problems . . . . .	87
	References . . . . .	89

<b>5</b>	<b>Stability and Suboptimality Using Stabilizing Terminal Conditions</b>	91
	5.1 The Relaxed Dynamic Programming Approach	91
	5.2 Equilibrium Endpoint Constraint	92
	5.3 Lyapunov Function Terminal Cost	99
	5.4 Suboptimality and Inverse Optimality	107
	5.5 Notes and Extensions	115
	Problems	116
	References	118
<b>6</b>	<b>Stability and Suboptimality Without Stabilizing Terminal Conditions</b>	121
	6.1 Setting and Preliminaries	121
	6.2 Bounds on $V_N$ and Asymptotic Controllability with Respect to $\ell$	125
	6.3 Implications of the Bound on $V_N$	129
	6.4 Computation of $\alpha$	130
	6.5 Main Stability and Performance Results	135
	6.6 Design of Good Stage Costs $\ell$	144
	6.7 Semiglobal and Practical Asymptotic Stability	154
	6.8 Proof of Proposition 6.18	163
	6.9 Notes and Extensions	172
	Problems	174
	References	176
<b>7</b>	<b>Feasibility and Robustness</b>	177
	7.1 The Feasibility Problem	177
	7.2 Feasibility of Unconstrained NMPC Using Exit Sets	180
	7.3 Feasibility of Unconstrained NMPC Using Stability	184
	7.4 Comparing NMPC with and Without Terminal Conditions	188
	7.5 Robustness: Basic Definition and Concepts	192
	7.6 Robustness Without State Constraints	194
	7.7 Examples for Nonrobustness Under State Constraints	199
	7.8 Robustness with State Constraints via Robust-Optimal Feasibility	204
	7.9 Robustness with State Constraints via Continuity of $V_N$	209
	7.10 Notes and Extensions	215
	Problems	217
	References	218
<b>8</b>	<b>Economic NMPC</b>	221
	8.1 Setting	221
	8.2 Averaged Performance with Terminal Conditions	223
	8.3 Asymptotic Stability with Terminal Conditions	227
	8.4 Non-averaged and Transient Performance with Terminal Conditions	231

8.5	Averaged Optimality Without Terminal Conditions . . . . .	239
8.6	Practical Asymptotic Stability Without Terminal Conditions . . . . .	243
8.7	Non-averaged and Transient Performance Without Terminal Conditions . . . . .	248
8.8	Notes and Extensions . . . . .	255
	Problems . . . . .	256
	References . . . . .	257
<b>9</b>	<b>Distributed NMPC . . . . .</b>	<b>259</b>
9.1	Background and Problem Formulation . . . . .	259
9.2	Classification of Connectedness . . . . .	261
9.3	Problem Classes for Different Levels of Connectedness . . . . .	272
9.4	Asymptotic Stability and Convergence . . . . .	276
9.5	Communication and Coordination Schemes . . . . .	281
9.6	Notes and Extensions . . . . .	292
	Problems . . . . .	293
	References . . . . .	294
<b>10</b>	<b>Variants and Extensions . . . . .</b>	<b>297</b>
10.1	Schemes with Mixed Terminal Conditions . . . . .	297
10.2	Unconstrained NMPC with Terminal Weights . . . . .	301
10.3	Nonpositive Definite Stage Cost . . . . .	302
10.4	Multistep NMPC-Feedback Laws . . . . .	306
10.5	Fast Sampling . . . . .	308
10.6	Compensation of Computation Times . . . . .	312
10.7	Online Measurement of $\alpha$ . . . . .	316
10.8	Adaptive Optimization Horizon . . . . .	325
10.9	Nonoptimal NMPC . . . . .	332
	References . . . . .	341
<b>11</b>	<b>Numerical Discretization . . . . .</b>	<b>343</b>
11.1	Basic Solution Methods . . . . .	343
11.2	Convergence Theory . . . . .	348
11.3	Adaptive Step Size Control . . . . .	353
11.4	Using the Methods Within the NMPC Algorithms . . . . .	357
11.5	Numerical Approximation Errors and Stability . . . . .	359
11.6	Notes and Extensions . . . . .	363
	Problems . . . . .	365
	References . . . . .	366
<b>12</b>	<b>Numerical Optimal Control of Nonlinear Systems . . . . .</b>	<b>367</b>
12.1	Discretization of the NMPC Problem . . . . .	367
12.2	Unconstrained Optimization . . . . .	380
12.3	Constrained Optimization . . . . .	385
12.4	Implementation Issues in NMPC . . . . .	408

12.5	Warm Start of the NMPC Optimization .....	418
12.6	Nonoptimal NMPC .....	426
12.7	Notes and Extensions .....	430
	Problems .....	432
	References .....	432
	<b>Appendix A: NMPC Software Supporting This Book .....</b>	<b>435</b>
	<b>Appendix B: Glossary .....</b>	<b>441</b>
	<b>Index .....</b>	<b>449</b>