

Advanced series on complex systems — volume 1

# **Theory and Applications of Cellular Automata**

(including selected papers 1983 — 1986)

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