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Ultrafast Lasers

A Comprehensive Introduction to
Fundamental Principles with Practical
Applications

 Springer

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