
Contents

	<i>Preface</i>	<i>page ix</i>
1	Prelude	1
	1.1 Presheaves	1
	1.2 The Category of Simplicial Sets	7
	1.3 Cellular Filtrations	9
	1.4 Nerves	14
	1.5 Definition of ∞ -Categories	19
	1.6 The Boardman–Vogt Construction	21
2	Basic Homotopical Algebra	27
	2.1 Factorisation Systems	28
	2.2 Model Categories	33
	2.3 Derived Functors	41
	2.4 Model Structures <i>ex Nihilo</i>	50
	2.5 Absolute Weak Equivalences	68
3	The Homotopy Theory of ∞-Categories	72
	3.1 Kan Fibrations and the Kan–Quillen Model Structure	73
	3.2 Inner Anodyne Extensions	88
	3.3 The Joyal Model Category Structure	92
	3.4 Left or Right Fibrations, Joins and Slices	98
	3.5 Invertible Natural Transformations	106
	3.6 ∞ -Categories as Fibrant Objects	112
	3.7 The Boardman–Vogt Construction, Revisited	116
	3.8 Serre’s Long Exact Sequence	120
	3.9 Fully Faithful and Essentially Surjective Functors	130
4	Presheaves: Externally	136
	4.1 Catégories Fibrées en ∞ -Groupoïdes	136
	4.2 Mapping Spaces as Fibres of Slices	144

4.3	Final Objects	149
4.4	Grothendieck Base Change Formulas and Quillen's Theorem A	155
4.5	Fully Faithful and Essentially Surjective Functors, Revisited	172
4.6	Locally Constant Functors and Quillen's Theorem B	175
5	Presheaves: Internally	183
5.1	Minimal Fibrations	184
5.2	The Universal Left Fibration	195
5.3	Homotopy Classification of Left Fibrations	204
5.4	Rectification of Morphisms	216
5.5	Bivariant Model Category Structures	225
5.6	The Twisted Diagonal	237
5.7	Locally Small ∞ -Categories	242
5.8	The Yoneda Lemma	248
6	Adjoints, Limits and Kan Extensions	260
6.1	Adjoints	261
6.2	Limits and Colimits	274
6.3	Extensions of Functors by Colimits	282
6.4	Kan Extensions	290
6.5	The Cartesian Product	296
6.6	Fibre Products	304
6.7	Duality	312
7	Homotopical Algebra	316
7.1	Localisation	319
7.2	Calculus of Fractions	328
7.3	Constructions of Limits	339
7.4	Finite Direct Diagrams	354
7.5	Derived Functors	366
7.6	Equivalences of ∞ -Categories with Finite Limits	382
7.7	Homotopy Completeness	392
7.8	The Homotopy Hypothesis	396
7.9	Homotopy Limits as Limits	402
7.10	Mapping Spaces in Locally Small Localisations	409
7.11	Presentable ∞ -Categories	413
	<i>Bibliography</i>	421
	<i>Notation</i>	426
	<i>Index</i>	428