

Contents

Preface	xi
Introduction	1
Preliminaries	17
Chapter 1. Coxeter groups and reflection arrangements	19
1.1. Face-types, flat-types, biface-types	20
1.2. Lune-types and bilune-types	29
1.3. Stabilizer subgroups of a Coxeter group	34
1.4. Gallery distance, gate property, length	35
1.5. Modules over the Coxeter group	37
1.6. Descents, global descents, shuffles	38
1.7. Groupoid of biface-types	40
1.8. Face-type enumeration	48
1.9. Enumeration in the Coxeter group algebra	60
1.10. Invariant flat-, lune-, bilune-incidence algebras	63
1.11. Noncommutative Zaslavsky formulas and q -analogues	75
1.12. Invariant Birkhoff, Tits, Janus algebras	77
1.13. Invariant Lie and Zie elements	84
1.14. Coxeter–Birkhoff, Coxeter–Tits, Coxeter–Janus algebras	85
1.15. Coordinate arrangement	87
1.16. Braid arrangement	90
1.17. Notes	95
Part I. Coxeter species	99
Chapter 2. Coxeter species and Coxeter bimonoids	101
2.1. Support and type morphisms	104
2.2. Induced and local morphisms	109
2.3. Coxeter species	110
2.4. Coxeter monoids and Coxeter comonoids	116
2.5. Coxeter bimonoids	122
2.6. (Co)commutative Coxeter (co)monoids	127
2.7. Species vs Coxeter species	135
2.8. Signature functors on Coxeter species	137
2.9. Coxeter (co, bi)monoids as functor categories	139

2.10.	Bimonads on Coxeter species	147
2.11.	Opposite transformation	153
2.12.	Joyal species and Coxeter species	154
2.13.	Notes	156
Chapter 3.	Basic theory of Coxeter bimonoids	159
3.1.	Cauchy powers of a Coxeter species	159
3.2.	Primitive and decomposable filtrations	160
3.3.	Universal constructions of Coxeter bimonoids	164
3.4.	(Co)abelianizations of Coxeter (co)monoids	174
3.5.	Hadamard product. Convolution Coxeter monoid	176
3.6.	Internal hom for Coxeter comonoids	180
3.7.	Universal measuring Coxeter comonoid	183
3.8.	Exp-log between (co)derivations and (co)monoid morphisms	185
3.9.	Logarithm of the identity map	190
3.10.	Exp-log between primitive and group-like series	193
3.11.	Norm transformation on Coxeter bimonoids	200
3.12.	Characteristic operations by faces	203
3.13.	Algebra modules and Coxeter bimonoids	205
3.14.	Antipodes of Coxeter bimonoids	210
3.15.	Notes	213
Chapter 4.	Examples of Coxeter bimonoids	216
4.1.	Coxeter species characteristic of chambers	216
4.2.	Exponential Coxeter bimonoid	218
4.3.	Coxeter bimonoid of chambers	222
4.4.	Coxeter bimonoid of flats	225
4.5.	Coxeter bimonoid of faces	228
4.6.	Coxeter bimonoid of top-nested faces	233
4.7.	Coxeter bimonoid of pairs of chambers	235
4.8.	Coxeter Lie species and Coxeter Zie species	237
4.9.	Decoration functor on Coxeter species	238
4.10.	Notes	244
Chapter 5.	Coxeter operads	246
5.1.	Coxeter dispecies	247
5.2.	Coxeter operads	248
5.3.	Hadamard product	252
5.4.	Coxeter operad monoids	252
5.5.	Coxeter commutative, associative, Lie operads	255
5.6.	Notes	260
Chapter 6.	Coxeter Lie monoids	261
6.1.	Coxeter Lie monoids and Coxeter Lie comonoids	262
6.2.	(Co)commutator (co)bracket	264
6.3.	Primitive and indecomposable parts of Coxeter bimonoids	265
6.4.	(Co)free Coxeter Lie (co)monoids	266

6.5.	Universal (co)enveloping Coxeter (co)monoids	269
6.6.	Joyal Lie monoids and Coxeter Lie monoids	271
6.7.	Notes	272
Chapter 7.	Structure theory of Coxeter bimonoids	273
7.1.	Borel–Hopf for (co)commutative Coxeter bimonoids	274
7.2.	Leray–Samelson for bicommutative Coxeter bimonoids	275
7.3.	Rigidity of Coxeter q -bimonoids for q not a root of unity	277
7.4.	Loday–Ronco for Coxeter 0-bimonoids	279
7.5.	Hoffman–Newman–Radford	280
7.6.	Poincaré–Birkhoff–Witt	284
7.7.	Cartier–Milnor–Moore	287
7.8.	Coxeter bimonoids for a rank-one arrangement	288
7.9.	Notes	297
Part II.	Coxeter spaces	299
Chapter 8.	Coxeter spaces and Coxeter bialgebras	301
8.1.	Support and type morphisms	304
8.2.	Induced and local morphisms	311
8.3.	Opposite and adjacent support-type morphisms	314
8.4.	Coxeter spaces	317
8.5.	Coxeter algebras and Coxeter coalgebras	324
8.6.	Coxeter bialgebras	331
8.7.	Coxeter q -bialgebras	341
8.8.	(Co)commutative Coxeter (co)algebras	347
8.9.	Coxeter bialgebras and face-type enumeration	355
8.10.	Signature functor on Coxeter spaces	356
8.11.	Coxeter (co, bi)algebras as functor categories	358
8.12.	Bimonads on Coxeter spaces	365
8.13.	Opposite transformation	372
8.14.	Graded vector spaces and Coxeter spaces	373
8.15.	Notes	379
Chapter 9.	Basic theory of Coxeter bialgebras	382
9.1.	Cauchy powers of a Coxeter space	383
9.2.	Primitive and decomposable filtrations	385
9.3.	Universal constructions of Coxeter bialgebras	388
9.4.	(Co)abelianizations of Coxeter (co)algebras	399
9.5.	Hadamard product. Convolution Coxeter algebra	403
9.6.	Internal hom for Coxeter coalgebras	406
9.7.	Universal measuring Coxeter coalgebra	409
9.8.	Exp-log between (co)derivations and (co)algebra morphisms	412
9.9.	Logarithm of the identity map	418
9.10.	Exp-log between primitive and group-like series	421
9.11.	Norm transformation on Coxeter bialgebras	430
9.12.	Bialgebra axiom on the primitive part	437

9.13.	Characteristic operations by face-types	439
9.14.	Antipodes of Coxeter bialgebras	445
9.15.	Notes	451
Chapter 10.	Examples of Coxeter bialgebras	456
10.1.	Coxeter space characteristic of chambers	457
10.2.	Coxeter bialgebra of polynomials	459
10.3.	Tensor, shuffle, symmetric, exterior Coxeter bialgebras	471
10.4.	Coxeter bialgebras of face-types	483
10.5.	Coxeter bialgebra of flat-types	505
10.6.	Coxeter bialgebra of symmetries	510
10.7.	Notes	515
Chapter 11.	Coxeter operad algebras	522
11.1.	Nested flats, nested top-lunes, nested face-types	523
11.2.	Coxeter dispecies	529
11.3.	Coxeter operads	531
11.4.	Coxeter spaces as a module over Coxeter dispecies	537
11.5.	Coxeter operad algebras	542
11.6.	Coxeter commutative, associative, Lie operads revisited	547
11.7.	Signature functor on Coxeter dispecies	554
11.8.	May operads and Coxeter operads	557
11.9.	Notes	560
Chapter 12.	Coxeter Lie algebras	561
12.1.	Coxeter Lie algebras and Coxeter Lie coalgebras	562
12.2.	(Co)commutator (co)bracket	567
12.3.	Primitive and indecomposable parts of Coxeter bialgebras	568
12.4.	(Co)free Coxeter Lie (co)algebras	571
12.5.	Universal (co)enveloping Coxeter (co)algebras	576
12.6.	Graded Lie algebras and Coxeter Lie algebras	582
12.7.	Notes	585
Chapter 13.	Structure theory of Coxeter bialgebras	586
13.1.	Borel–Hopf for (co)commutative Coxeter bialgebras	588
13.2.	Leray–Samelson for bicommutative Coxeter bialgebras	595
13.3.	Rigidity of Coxeter q -bialgebras for q not a root of unity	600
13.4.	Loday–Ronco for Coxeter 0-bialgebras	606
13.5.	Hoffman–Newman–Radford	607
13.6.	Poincaré–Birkhoff–Witt	614
13.7.	Cartier–Milnor–Moore	620
13.8.	Coxeter bialgebras for a rank-one arrangement	623
13.9.	Notes	638
Part III.	Fock functors	641
Chapter 14.	Fock functors	643
14.1.	Full Fock functors	644

14.2.	Deformed full Fock functors	651
14.3.	Full Fock functors and signature functors	653
14.4.	Bosonic and fermionic Fock functors	656
14.5.	Full Fock functors from bosonic Fock functors	661
14.6.	Fock functors and commutativity	662
14.7.	Norm transformation between Fock functors	671
14.8.	Anyonic Fock functor	674
14.9.	Fock functors and Coxeter operads	676
14.10.	Decorated Fock functors	682
14.11.	Notes	684
Chapter 15.	Coxeter bimonoids and Coxeter bialgebras	687
15.1.	Coxeter bimonoids to Coxeter bialgebras	689
15.2.	Commutativity under Fock functors	694
15.3.	Norm map between Fock spaces	695
15.4.	(Co)freeness under Fock functors	697
15.5.	Nonnegativity of the Boolean transform	701
15.6.	Coxeter operad monoids to Coxeter operad algebras	702
15.7.	Coxeter Lie monoids to Coxeter Lie algebras	705
15.8.	Primitive and indecomposable parts under Fock functors	708
15.9.	Universal enveloping constructions under Fock functors	710
15.10.	Notes	712
Chapter 16.	Adjoints of Fock functors	714
16.1.	Trivialization functor	717
16.2.	Hadamard product with chambers	721
16.3.	Adjoints of bosonic Fock functors	722
16.4.	Lifts of adjunctions involving bosonic Fock functors	725
16.5.	Bosonic Fock functors and abelianization	739
16.6.	Adjoints preserving commutativity	740
16.7.	Adjoints of fermionic Fock functors	745
16.8.	Adjoints of full Fock functors	745
16.9.	Coxeter operad algebras to Coxeter operad monoids	750
16.10.	Coxeter Lie algebras to Coxeter Lie monoids	753
16.11.	(Co)freeness under Fock functor adjoints	756
16.12.	Notes	757
Chapter 17.	Structure theory under Fock functors	759
17.1.	Series of Coxeter species and Coxeter spaces	759
17.2.	Actions of the invariant bilune-incidence algebra	762
17.3.	Internal hom for Coxeter species and Coxeter spaces	766
17.4.	Norm maps on Coxeter species and Coxeter spaces	769
17.5.	Transformation between bosonic Fock functor adjoints	772
17.6.	Antipodes under Fock functors	776
17.7.	Structure theorems under Fock functors	778
17.8.	Lie theory under Fock functors	782
17.9.	Notes	785

Chapter 18. Examples of Fock spaces	786
18.1. Coxeter space characteristic of chambers as a Fock space	787
18.2. Coxeter bialgebra of polynomials as a Fock space	788
18.3. Tensor Coxeter bialgebra as a Fock space	794
18.4. Coxeter bialgebra of face-types as a bosonic Fock space	798
18.5. Coxeter bialgebra of flat-types as a bosonic Fock space	804
18.6. Coxeter bialgebra of symmetries as a bosonic Fock space	806
18.7. Fock functors for a rank-one arrangement	807
18.8. Notes	815
Appendix A. Category theory	816
A.1. Bimonads and bilax functors	816
A.2. Semidirect products	821
A.3. Notes	824
References	826
List of Notations	842
List of Tables	852
List of Figures	855
List of Summaries	857
Author Index	858
Subject Index	863