Contents

1	Introduction				
	1.1	Ellipsoids and Hyperboloids	2		
		Intersections of Two Concentric Ellipsoids			
	1.3	Intersections of Two Concentric Hyperboloids	5		
	1.4	Plan of the Book	9		

Part I General Results

2	General Intersections of Quadrics				
	2.1	Chara	cterization of General Transverse Compact Intersections of		
		Quadr	ics	17	
	2.2	Chara	cterization of General Transverse Intersections of Ellipsoids	20	
	2.3	Quadr	atic Mappings	22	
	2.4	Types	of Intersections, Their Symmetries and Operations	23	
		2.4.1	Intersections of concentric quadrics	24	
		2.4.2	Intersections of concentric ellipsoids	25	
		2.4.3	Universality of open half intersections of concentric		
			ellipsoids	27	
		2.4.4	Intersections of partially coaxial quadrics	30	
		2.4.5	Intersections of coaxial quadrics	32	
		2.4.6	Intersections of coaxial ellipsoids	33	
		2.4.7	Moment-angle manifolds	35	
3	General Operations on Intersections of Quadrics			39	
	3.1	The B	ook Construction	39	
	3.2	Addin	g Squares and Operation \tilde{Y}	44	
		3.2.1	Adding a real square	44	
		3.2.2	Adding a complex square	48	
		3.2.3	Operation \widetilde{Y}		

4	Inte	rsection	ns of Coaxial Quadrics	57
	4.1	Symm	etry and the Polyhedral Set	57
	4.2	Gener	al Properties	61
		4.2.1	Non-emptiness	61
		4.2.2	Compactness	62
		4.2.3	Transversality	63
		4.2.4	Transversality at infinity	66
		4.2.5	Transversality up to infinity	67
		4.2.6	Connectedness	68
		4.2.7	Simple connectedness	70
		4.2.8	Higher connectedness	72
	4.3	Polyhe	edral Sets and Polytopes. Realization and Operations	73
		4.3.1	Realization	73
	4.4	Trunca	ation	77
		4.4.1	Truncating faces	79
		4.4.2	Truncating vertices	81
		4.4.3	Truncating simplicial faces	83
		4.4.4	Doubles, open books and connected sums	85
		4.4.5	Combining truncations with the book construction	87
5	Inte	rsection	ns of Coaxial Ellipsoids	91
	5.1		al Properties of Intersections of Coaxial Ellipsoids	92
		5.1.1	Non-emptiness	92
		5.1.2	Transversality	93
		5.1.3	Connectedness	94
		5.1.4	Higher connectedness	94
		5.1.5	General properties of moment-angle manifolds	95
	5.2	Primit	ive Configurations and Multiplicities	
		5.2.1	Some small configurations	99
	5.3	Trunca	ation of Transverse Intersections of Coaxial Ellipsoids	103
		5.3.1	Some simple vertex truncations	
		5.3.2	Truncating faces of a product of simplices	
		5.3.3	A deeper cut	
	5.4	The D	ual Polytope	
	5.5		ar Intersections	
		5.5.1	Cones and their smoothings	
		5.5.2	Some singular intersections	
		5.5.3	The link of an isolated singularity	
		5.5.4	Codimension one singularities	
		5.5.5	Smoothings and wall-crossing	
	5.6		omology Splitting	
	5.7		ples of Homology Computations	
	2.1	5.7.1	Examples of homology computations of singular	
			intersections	130

5.8	Dualities	131
5.9	The Sphere and Singularity Theorems	134

Part II Topological Description of Transverse Intersections of Concentric Ellipsoids

6	Cha	racterization of Connected Sums
7	Thr	ee Coaxial Ellipsoids
1011	7.1	Main Theorem 7.1
	7.2	The Homology for Three Coaxial Ellipsoids
	7.3	Proof of the Main Theorem 7.1
	7.4	Parallelizability and Euler Characteristic
	7.5	Halves
	1.0	7.5.1 The space $\mathcal{E}_{p,q}^{m}$
		7.5.2 The topology of Z_+
	7.6	Transverse Intersections of Two Coaxial Hyperboloids
	1.0	
8	Thr	ee Concentric Ellipsoids
	8.1	The Normal Form
		8.1.1 Normal form of two complex homogeneous quadrics 158
		8.1.2 Linear normal form of two homogeneous real quadrics 161
		8.1.3 Topological normal form of three transverse concentric
		ellipsoids
	8.2	The Main Theorem 8.5 165
	8.3	Preservation of Connected Sums
	8.4	Homology
		8.4.1 Preservation of the total homology
		8.4.2 Computation of the homology
	8.5	Proof of the Main Theorem 8.5 170
9	Mor	re Than Three Coaxial Ellipsoids
,	9.1	Dual-Neighborly Polytopes
	9.2	The Topology of the Associated Intersection of Coaxial Ellipsoids . 177
	1.2	9.2.1 The Euler characteristic $\chi(Z(P))$ for dual-neighborly
		polytopes P of even dimension
		9.2.2 The result of other operations
		9.2.3 On the sequences of genera
		5.2.5 On the sequences of general first first first first first first
10	A F	amily of Surfaces That Are Intersections of Concentric,
		-Coaxial Ellipsoids
		Actions of 2-Groups on Surfaces with Quotient a Polygon 185
		The Construction
	10.3	Proof of Theorem 10.1 of the Previous Section

Part III Relations With Other Areas of Mathematics

11	Dynamical Systems
	11.1 Real Linear Dynamical Systems
	11.2 Linear Complex Dynamical Systems
	11.3 Generalized Hopf bifurcations
	11.4 Generalized May–Leonard Systems
12	Complex Geometry
12	12.1 The Main Classical Examples
	12.2 Deformations of the Main Classical Examples
	12.2 Deformations of the Main Classical Examples
	12.4 Deformations
	12.5 Examples
13	Contact and Symplectic Geometry
	13.1 All Odd-Dimensional Moment-Angle Manifolds Admit Contact
	Structures
	13.2 Large Families of Odd-Dimensional Coaxial Intersections of Ellipsoids Admit Contact Structures
	13.3 A Family of Odd-Dimensional Concentric Intersections of
	Ellipsoids That Admit Contact Structures
	13.4 Intersections of Ellipsoids as Lagrangian Submanifolds 213
14	Intersections with Dihedral Symmetry
	14.1 Jacobi Formula for the Co-Rank
	14.2 Minors of the Vandermonde V_n on the <i>n</i> -th Roots of Unity
	14.2.1 First results
	14.2.2 The complementarity theorem
	14.2.3 The case $n = p$ prime. Chebotaryov's Theorem
	14.2.4 Some cases where n is a prime power, $n = p^k$, p odd 221
	14.2.5 The Murty–Whang criterion
	14.3 Some Complex Varieties With Cyclic and Dihedral Symmetry 224
	14.3.1 Some complex varieties with cyclic symmetry
	14.3.2 Some complex varieties with dihedral symmetry
	14.4 Intersections of Real Varieties With Dihedral Symmetry 228
15	Polyhedral Products

Part IV Appendices

A	Proo	f of Theorem 2.1		
B	Orig	Origins		
	B.1	From Singularity Theory		
	B.2	Dynamical Systems		
	B.3	to the Polyhedral Product Functor		
		B.3.1 Coxeter groups, small covers and toric manifolds		
		B.3.2 The polyhedral product functor		
С	Com	plements of Products of Spheres in Spheres		
D	Diag	onalizability of Matrices		
	D.1	Generalities		
	D.2	When the Field is \mathbb{R} or \mathbb{C}		
	D.3	Simultaneous Diagonalizability and Commutation		
	D.4	An Algorithm for Simultaneous Diagonalizability		
	D.5	Some Algebraic Mappings Between Spaces of Matrices		
Refe	erence	es		
Ind	ex			