
Contents

Preface	vii
Acknowledgments	ix
1 Introduction	1
1.1 From lines and planes to the Zariski topology of \mathbb{P}^n	1
1.2 The main theorem	6
1.3 Organization of the book	8
2 Preliminaries	11
2.1 Algebraic varieties	11
2.2 Examples and speculations	13
2.3 Scheme-theoretic formulation	17
2.4 Survey of related results	18
2.5 Terminology and notation	21
3 The fundamental theorem of projective geometry	25
3.1 The fundamental theorem of definable projective geometry	25
3.2 A variant fundamental theorem	34
3.3 The probabilistic fundamental theorem of projective geometry . .	39
4 Divisorial structures and definable linear systems	55
4.1 Divisorial structures	55
4.2 Remarks on divisors	59
4.3 Definable subspaces in linear systems	62
5 Reconstruction from divisorial structures: infinite fields	68
5.1 Reduction to the quasi-projective case	69
5.2 The quasi-projective case	70
5.3 Counterexamples in dimension 1	76
6 Reconstruction from divisorial structures: finite fields	79
6.1 The Bertini-Poonen theorem	80
6.2 Preparatory lemmas	83
6.3 Reconstruction over finite fields	86

7	Topological geometry	94
7.1	Pencils	95
7.2	Fibers of finite morphisms	103
7.3	Topological pencils	106
7.4	Degree functions and algebraic pencils	109
7.5	Degree functions and linear equivalence	112
7.6	Uncountable fields	116
8	The set-theoretic complete intersection property	117
8.1	Summary of results	117
8.2	Set-theoretic complete intersection property	118
8.3	Mordell-Weil fields	123
8.4	Reducible scip subsets	126
8.5	Projective spaces	132
8.6	Appendix: Special fields	134
9	Linkage	139
9.1	Linkage of divisors	140
9.2	Preparations: Sections and their zero sets	141
9.3	Néron's theorem and consequences	144
9.4	Linear similarity	146
9.5	Bertini-Hilbert dimension	152
9.6	Linkage of divisors and residue fields	154
9.7	Minimally restrictive linking and transversality	159
9.8	Recovering linear equivalence	163
9.9	Appendix: Weakly Hilbertian fields	168
10	Complements, counterexamples, and conjectures	177
10.1	A topological Gabriel theorem	177
10.2	Examples over finite fields	179
10.3	Surfaces over locally finite fields	182
10.4	Real Zariski topology	182
10.5	Countable noetherian topologies	183
10.6	Conjectures	186
11	Appendix	190
11.1	Bertini-type theorems	190
11.2	Complete intersections	194
11.3	Picard group, class group, and Albanese variety	199
	Bibliography	213
	Index of Notation	223
	Index of Terminology	225