
Contents

Preface	xv
I Introduction and Classical Cryptography	
1 Introduction	1
1.1 Cryptography and Modern Cryptography	1
1.2 The Setting of Private-Key Encryption	2
1.3 Historical Ciphers and Their Cryptanalysis	6
1.4 Principles of Modern Cryptography	14
1.4.1 Principle 1 – Formal Definitions	15
1.4.2 Principle 2 – Precise Assumptions	18
1.4.3 Principle 3 – Proofs of Security	20
1.4.4 Provable Security and Real-World Security	20
References and Additional Reading	21
Exercises	21
2 Perfectly Secret Encryption	23
2.1 Definitions	24
2.2 The One-Time Pad	31
2.3 Limitations of Perfect Secrecy	33
2.4 *Shannon’s Theorem	34
References and Additional Reading	36
Exercises	36
II Private-Key (Symmetric) Cryptography	41
3 Private-Key Encryption	43
3.1 Computational Security	43
3.1.1 The Concrete Approach	44
3.1.2 The Asymptotic Approach	45
3.2 Defining Computationally Secure Encryption	51
3.2.1 The Basic Definition of Security (EAV-Security)	52
3.2.2 *Semantic Security	56
3.3 Constructing an EAV-Secure Encryption Scheme	60
3.3.1 Pseudorandom Generators	60
3.3.2 Proofs by Reduction	64
3.3.3 EAV-Security from a Pseudorandom Generator	65

3.4	Stronger Security Notions	70
3.4.1	Security for Multiple Encryptions	70
3.4.2	Chosen-Plaintext Attacks and CPA-Security	72
3.4.3	CPA-Security for Multiple Encryptions	74
3.5	Constructing a CPA-Secure Encryption Scheme	75
3.5.1	Pseudorandom Functions and Permutations	76
3.5.2	CPA-Security from a Pseudorandom Function	80
3.6	Modes of Operation and Encryption in Practice	84
3.6.1	Stream Ciphers	85
3.6.2	Stream-Cipher Modes of Operation	87
3.6.3	Block Ciphers and Block-Cipher Modes of Operation	88
3.6.4	*Nonce-Based Encryption	96
	References and Additional Reading	99
	Exercises	99
4	Message Authentication Codes	105
4.1	Message Integrity	105
4.1.1	Secrecy vs. Integrity	105
4.1.2	Encryption vs. Message Authentication	106
4.2	Message Authentication Codes (MACs) – Definitions	108
4.3	Constructing Secure Message Authentication Codes	114
4.3.1	A Fixed-Length MAC	114
4.3.2	Domain Extension for MACs	116
4.4	CBC-MAC	120
4.4.1	The Basic Construction	120
4.4.2	*Proof of Security	123
4.5	GMAC and Poly1305	128
4.5.1	MACs from Difference-Universal Functions	128
4.5.2	Instantiations	131
4.6	*Information-Theoretic MACs	133
4.6.1	One-Time MACs from Strongly Universal Functions	134
4.6.2	One-Time MACs from Difference-Universal Functions	137
4.6.3	Limitations on Information-Theoretic MACs	139
	References and Additional Reading	140
	Exercises	140
5	CCA-Security and Authenticated Encryption	145
5.1	Chosen-Ciphertext Attacks and CCA-Security	145
5.1.1	Padding-Oracle Attacks	146
5.1.2	Defining CCA-Security	149
5.2	Authenticated Encryption	151
5.2.1	Defining Authenticated Encryption	151
5.2.2	CCA Security vs. Authenticated Encryption	153
5.3	Authenticated Encryption Schemes	154
5.3.1	Generic Constructions	154

5.3.2	Standardized Schemes	161
5.4	Secure Communication Sessions	162
	References and Additional Reading	164
	Exercises	164
6	Hash Functions and Applications	167
6.1	Definitions	167
6.1.1	Collision Resistance	168
6.1.2	Weaker Notions of Security	170
6.2	Domain Extension: The Merkle–Damgård Transform	170
6.3	Message Authentication Using Hash Functions	172
6.3.1	Hash-and-MAC	172
6.3.2	HMAC	175
6.4	Generic Attacks on Hash Functions	177
6.4.1	Birthday Attacks for Finding Collisions	178
6.4.2	Small-Space Birthday Attacks	179
6.4.3	*Time/Space Tradeoffs for Inverting Hash Functions	182
6.5	The Random-Oracle Model	187
6.5.1	The Random-Oracle Model in Detail	188
6.5.2	Is the Random-Oracle Methodology Sound?	192
6.6	Additional Applications of Hash Functions	195
6.6.1	Fingerprinting and Deduplication	195
6.6.2	Merkle Trees	196
6.6.3	Password Hashing	198
6.6.4	Key Derivation	199
6.6.5	Commitment Schemes	200
	References and Additional Reading	202
	Exercises	203
7	Practical Constructions of Symmetric-Key Primitives	207
7.1	Stream Ciphers	208
7.1.1	Linear-Feedback Shift Registers	209
7.1.2	Adding Nonlinearity	211
7.1.3	Trivium	212
7.1.4	RC4	213
7.1.5	ChaCha20	216
7.2	Block Ciphers	217
7.2.1	Substitution-Permutation Networks	219
7.2.2	Feistel Networks	226
7.2.3	DES – The Data Encryption Standard	228
7.2.4	3DES: Increasing the Key Length of a Block Cipher	235
7.2.5	AES – The Advanced Encryption Standard	238
7.2.6	*Differential and Linear Cryptanalysis	240
7.3	Compression Functions and Hash Functions	246
7.3.1	Compression Functions from Block Ciphers	246

7.3.2	MD5, SHA-1, and SHA-2	249
7.3.3	The Sponge Construction and SHA-3 (Keccak)	250
	References and Additional Reading	254
	Exercises	255
8	*Theoretical Constructions of Symmetric-Key Primitives	261
8.1	One-Way Functions	262
8.1.1	Definitions	262
8.1.2	Candidate One-Way Functions	265
8.1.3	Hard-Core Predicates	266
8.2	From One-Way Functions to Pseudorandomness	267
8.3	Hard-Core Predicates from One-Way Functions	269
8.3.1	A Simple Case	270
8.3.2	A More Involved Case	270
8.3.3	The Full Proof	274
8.4	Constructing Pseudorandom Generators	277
8.4.1	Pseudorandom Generators with Minimal Expansion	277
8.4.2	Increasing the Expansion Factor	279
8.5	Constructing Pseudorandom Functions	284
8.6	Constructing (Strong) Pseudorandom Permutations	289
8.7	Assumptions for Private-Key Cryptography	293
8.8	Computational Indistinguishability	296
	References and Additional Reading	298
	Exercises	299
III	Public-Key (Asymmetric) Cryptography	303
9	Number Theory and Cryptographic Hardness Assumptions	305
9.1	Preliminaries and Basic Group Theory	306
9.1.1	Primes and Divisibility	307
9.1.2	Modular Arithmetic	309
9.1.3	Groups	311
9.1.4	The Group \mathbb{Z}_N^*	315
9.1.5	*Isomorphisms and the Chinese Remainder Theorem	317
9.2	Primes, Factoring, and RSA	322
9.2.1	Generating Random Primes	323
9.2.2	*Primality Testing	325
9.2.3	The Factoring Assumption	331
9.2.4	The RSA Assumption	331
9.2.5	*Relating the Factoring and RSA Assumptions	334
9.3	Cryptographic Assumptions in Cyclic Groups	336
9.3.1	Cyclic Groups and Generators	336
9.3.2	The Discrete-Logarithm/Diffie–Hellman Assumptions	339
9.3.3	Working in (Subgroups of) \mathbb{Z}_p^*	342
9.3.4	Elliptic Curves	345

9.4	*Cryptographic Applications	354
9.4.1	One-Way Functions and Permutations	355
9.4.2	Collision-Resistant Hash Functions	357
	References and Additional Reading	359
	Exercises	360
10	*Algorithms for Factoring and Computing Discrete Logarithms	365
10.1	Algorithms for Factoring	366
10.1.1	Pollard's $p - 1$ Algorithm	367
10.1.2	Pollard's Rho Algorithm	368
10.1.3	The Quadratic Sieve Algorithm	369
10.2	Generic Algorithms for Computing Discrete Logarithms	372
10.2.1	The Pohlig–Hellman Algorithm	374
10.2.2	The Baby-Step/Giant-Step Algorithm	376
10.2.3	Discrete Logarithms from Collisions	377
10.3	Index Calculus: Computing Discrete Logarithms in \mathbb{Z}_p^*	378
10.4	Recommended Key Lengths	380
	References and Additional Reading	381
	Exercises	382
11	Key Management and the Public-Key Revolution	385
11.1	Key Distribution and Key Management	385
11.2	A Partial Solution: Key-Distribution Centers	387
11.3	Key Exchange and the Diffie–Hellman Protocol	389
11.4	The Public-Key Revolution	396
	References and Additional Reading	398
	Exercises	399
12	Public-Key Encryption	401
12.1	Public-Key Encryption – An Overview	401
12.2	Definitions	404
12.2.1	Security against Chosen-Plaintext Attacks	405
12.2.2	Multiple Encryptions	407
12.2.3	Security against Chosen-Ciphertext Attacks	412
12.3	Hybrid Encryption and the KEM/DEM Paradigm	415
12.3.1	CPA-Security	419
12.3.2	CCA-Security	424
12.4	CDH/DDH-Based Encryption	425
12.4.1	El Gamal Encryption	426
12.4.2	DDH-Based Key Encapsulation	430
12.4.3	*A CDH-Based KEM in the Random-Oracle Model	432
12.4.4	*Chosen-Ciphertext Security and DHIES/ECIES	434
12.5	RSA-Based Encryption	436
12.5.1	Plain RSA Encryption	436

12.5.2	Padded RSA and PKCS #1 v1.5	441
12.5.3	*CPA-Secure Encryption without Random Oracles	443
12.5.4	OAEP and PKCS #1 v2	447
12.5.5	*A CCA-Secure KEM in the Random-Oracle Model	451
12.5.6	RSA Implementation Issues and Pitfalls	455
	References and Additional Reading	458
	Exercises	459
13	Digital Signature Schemes	463
13.1	Digital Signatures – An Overview	463
13.2	Definitions	465
13.3	The Hash-and-Sign Paradigm	467
13.4	RSA-Based Signatures	468
13.4.1	Plain RSA Signatures	468
13.4.2	RSA-FDH and PKCS #1 Standards	470
13.5	Signatures from the Discrete-Logarithm Problem	475
13.5.1	Identification Schemes and Signatures	475
13.5.2	The Schnorr Identification/Signature Schemes	480
13.5.3	DSA and ECDSA	483
13.6	Certificates and Public-Key Infrastructures	485
13.7	Putting It All Together – TLS	491
13.8	*Signcryption	493
	References and Additional Reading	495
	Exercises	495
14	*Post-Quantum Cryptography	499
14.1	Post-Quantum Symmetric-Key Cryptography	500
14.1.1	Grover’s Algorithm and Symmetric-Key Lengths	500
14.1.2	Collision-Finding Algorithms and Hash Functions	501
14.2	Shor’s Algorithm and its Impact on Cryptography	502
14.3	Post-Quantum Public-Key Encryption	504
14.4	Post-Quantum Signatures	509
14.4.1	Lamport’s Signature Scheme	510
14.4.2	Chain-Based Signatures	513
14.4.3	Tree-Based Signatures	517
	References and Additional Reading	522
	Exercises	523
15	*Advanced Topics in Public-Key Encryption	525
15.1	Public-Key Encryption from Trapdoor Permutations	525
15.1.1	Trapdoor Permutations	526
15.1.2	Public-Key Encryption from Trapdoor Permutations	527
15.2	The Paillier Encryption Scheme	529
15.2.1	The Structure of $\mathbb{Z}_{N^2}^*$	530
15.2.2	The Paillier Encryption Scheme	532

	xiii
15.2.3 Homomorphic Encryption	537
15.3 Secret Sharing and Threshold Encryption	539
15.3.1 Secret Sharing	539
15.3.2 Verifiable Secret Sharing	541
15.3.3 Threshold Encryption and Electronic Voting	543
15.4 The Goldwasser–Micali Encryption Scheme	545
15.4.1 Quadratic Residues Modulo a Prime	545
15.4.2 Quadratic Residues Modulo a Composite	548
15.4.3 The Quadratic Residuosity Assumption	552
15.4.4 The Goldwasser–Micali Encryption Scheme	553
15.5 The Rabin Encryption Scheme	556
15.5.1 Computing Modular Square Roots	556
15.5.2 A Trapdoor Permutation Based on Factoring	561
15.5.3 The Rabin Encryption Scheme	565
References and Additional Reading	566
Exercises	567
Index of Common Notation	571
Appendix A Mathematical Background	575
A.1 Identities and Inequalities	575
A.2 Asymptotic Notation	575
A.3 Basic Probability	576
A.4 The “Birthday” Problem	581
A.5 *Finite Fields	584
Appendix B Basic Algorithmic Number Theory	587
B.1 Integer Arithmetic	589
B.1.1 Basic Operations	589
B.1.2 The Euclidean and Extended Euclidean Algorithms	590
B.2 Modular Arithmetic	591
B.2.1 Basic Operations	592
B.2.2 Computing Modular Inverses	592
B.2.3 Modular Exponentiation	593
B.2.4 *Montgomery Multiplication	595
B.2.5 Choosing a Uniform Group Element	597
B.3 *Finding a Generator of a Cyclic Group	599
B.3.1 Group-Theoretic Background	599
B.3.2 Efficient Algorithms	601
References and Additional Reading	602
Exercises	602
References	603
Index	619