CONTENTS

PART ONE INTRODUCTORY TOPICS IN OPERATIONS RESEARCH / 1

1. An Overview of Operations Research / 3
1.1 The Decision-Making Process / 4
1.2 The Nature of Operations Research / 7
1.3 Limitations of Operations Research / 11
1.4 Acceptance and Implementation of Operations Research / 13
1.5 Summary / 15

2. An Examination of Models as They Relate to Operations Research / 17
2.1 The Concept of a Model and a Classification Scheme / 18
2.2 Constructing a Model / 21
2.3 Validating a Model / 23
2.4 Solution Algorithms / 26
2.5 Models That Have General Usefulness / 27
2.6 Summary / 34

3. The Basics of Linear Programming / 35
3.1 The General Model / 36
3.2 Defining the Decision Variables / 39
3.3 Some Basic Examples / 40
3.4 Assumptions of Linear Programming / 46
3.5 Solution by the Graphical Method / 46
3.6 Summary / 51 Problems / 51

4. An Interpretive Tour Through a Linear-Programming Solution / 61
4.1 *Statement of an Illustrative Problem / 61*
4.2 *Definitions / 62*
4.3 *The Solution Process / 63*
4.4 *Applying the Solution Process / 64*
4.4A—*The First BFS / 65*
4.4B—*The First Set of* q_i *Values / 66*
4.4C—*The First Set of* z_j *Values and the Optimality Test / 66*
4.4D—*Selecting the Entering and Departing Variables / 66*
4.4E—*The New BFS / 68*
4.4F—*The Second Set of* q_i *Values / 68*
4.4G—*The Optimality Test / 70*
4.4H—*Finding the New BFS / 70*
4.4I—*The Final Test / 71*
4.5 *Summary of the Mathematics / 73*
4.6 *Extensions and Summary / 75* *Problems / 77*

5. The Simplex Method / 81
5.1 *The General Maximization Problem with Less-than-or-Equal-to Constraints / 82*
5.2 *The Simplex Format / 83*
5.3 *The Simplex Rules / 85*
5.4 *Minimization Problems and Greater-than-or-Equal-to Constraints / 90*
5.4A—*The Big M Method / 91*
5.4B—*The Two-Phase Method / 92*
5.5 *Equality Constraints / 94*
5.6 *Degeneracy / 94*
5.7 *Nonfeasibility / 96*
5.8 *Unbounded Solutions / 97*
5.9 *Multiple Solutions / 98*
5.10 *Variables Unrestrained in Sign / 99*
5.11 *Summary / 100*
Appendix—A Linear Programming Computer Code / 101
Problems / 111

6. Selected Case Studies in Linear Programming / 115
6.1 *The Blending Problem / 116*
6.2 *The Joint-Products Problem / 119*
6.3 *A Multiperiod Problem / 121*
6.4 *Production Scheduling / 123*
6.5 *The Trim-Loss Problem / 124*
6.6 *The Shrinkage Problem / 127*
6.7 *A Trilevel Decision Problem / 128*
6.8 *Summary / 130* *Problems / 131*

7. Sensitivity Analysis of Linear Programming / 141
7.1 The Marginal Values / 142
7.2 Analysis of p_j; j Not in the Basis / 144
7.3 Analysis of p_j; j in the Basis / 145
7.4 Case Study Showing Potential Uses of Sensitivity Analyses / 148
7.5 Analysis of b_i / 152
7.6 Analysis of the $a_{i,j}$ Values / 156
7.7 New Variables / 159
7.8 Continuation of the Case Study from Section 7.4 / 160
7.9 Summary / 163 *Problems / 165*

8. The Dual and Dual Simplex Method / 177
8.1 The Concept of the Dual / 178
8.2 The Dual: An Interpretation / 180
8.3 Using the Dual / 183
8.4 Resolving Some Complications / 185
8.5 The Dual Simplex Method / 188
8.6 An Application of the Dual Simplex Method: Adding Constraints / 192
8.7 Summary / 195
Appendix—Proof of the Primal-Dual Relationships / 196
Problems / 202

9. Parametric Programming / 207
9.1 The General Nature of Parametric Programming / 208
9.2 Partial Ranging of a Parametric Objective Function / 210
9.3 An Algorithm for the Complete Ranging of a Parametric Objective Function / 214
9.4 Partial Ranging of a Parametric B Vector / 219
9.5 An Algorithm for the Complete Ranging of a Parametric B Vector / 221
9.6 Ad Hoc Parametric Functions / 227
9.7 Summary / 228 *Problems / 228*

10. The Transportation and Related Algorithms / 233
10.1 The Transportation Problem / 234
10.1A—The General Transportation Problem / 234
10.1B—Finding the Initial BFS / 235
10.1C—Testing for Optimality / 240
10.1D—Pivoting to a New BFS / 244
10.1E—Problem Completion / 245
10.1F—Pivoting Revisited / 246
10.1G—An Unbalanced Transportation Problem / 247
10.1H—Degeneracy / 248
10.1I—Extensions of the Transportation Problem / 250

10.2 The Transshipment Problem / 252
10.3 The Assignment Problem / 254
10.3A—The Effectiveness Matrix / 255
10.3B—The Initial-Assignment Procedure / 256
10.3C—The Improvement Routine / 257
10.4 Summary / 261 Problems / 262

11. PERT / 271
11.1 The PERT Network / 272
11.2 Finding the Critical Path / 275
11.3 Float Times: Their Meaning and Computation / 279
11.4 Uncertainty / 281
11.5 Resource Allocation in CPM / 286
11.6 Summary / 293 Problems / 294

12. Deterministic Inventory Models / 301
12.1 The Simple EOQ: A Specific Example / 303
12.2 The General Model for the Simple EOQ / 309
12.3 The Production Model / 312
12.4 The Case with Two Levels of Storage / 315
12.5 Reorder Point and Safety Stock / 318
12.6 Summary / 322 Problems / 323

13. Additional Topics in Deterministic Inventory Theory / 331
13.1 The Stockout Model / 332
13.2 Sensitivity Analysis of Inventory Models / 338
13.3 EOQ Models with Quantity Discounts / 344
13.3A—The All-Units-Discount Method / 344
13.3B—The Incremental-Quantity-Discount Method / 350
13.4 Summary / 354 Problems / 355

PART TWO INTERMEDIATE TOPICS IN MATHEMATICAL PROGRAMMING / 363

14. Goal Programming / 365
14.1 Introduction to Goal Programming / 366
14.2 The Single-Goal Model / 367
14.3 The Multigoal Model: Goals and Deviations Equally Ranked / 368
14.4 Multigoals: Priority Ranking of Goals / 370
14.5 Multigoals: Nonadditivity of Goal Deviations / 372
14.6 Summary of the Model and the Solution Algorithm / 373
14.7 Applications of Goal Programming / 376
14.8 Summary / 383 Problems / 384

15. Topics in Network Theory / 391
15.1 Definitions / 392
15.2 The Shortest-Route Algorithm / 393
15.3 The Maximal-Flow Problem / 397
15.4 Applications that Require the Determination of a Feasible Flow at a Minimum Cost / 404
15.5 The Out-of-Kilter Algorithm / 407
15.6 Summary / 419 Problems / 420

16. Integer Programming / 425
16.1 Introduction to Integer Programming via Graphical Techniques / 426
16.2 Cases that Require Integer Solutions / 428
16.2A—A Simple Case / 428
16.2B—Capital Budgeting / 429
16.2C—Either-or Constraints / 430
16.2D—The Case Where at Least k of p Constraints Must Hold / 432
16.2E—Step Fixed Costs / 433
16.2F—Plant Location / 435
16.2G—The Traveling-Salesman Problem / 436
16.2H—Selecting Advertising Media / 439
16.2I—The Machine-Scheduling Problem / 441
16.3 Summary / 444 Problems / 445

17. Selected Integer Programming Algorithms / 455
17.1 The All-Integer Algorithm / 456
17.2 The Mixed-Integer Algorithm / 463
17.3 The Branch-and-Bound Algorithm for Either an All- or a Mixed-Integer Problem / 466
17.4 The Zero-One Algorithm / 473
17.5 Summary / 482 Problems / 483

18. Dynamic Programming / 487
18.1 The Nature of Dynamic Programming / 490
18.2 The Notation, Definitions, and General Model / 493
18.3 The "Knapsack" Problem / 496
18.4 A Production Problem / 500
18.5 A Reliability Problem / 503
18.6 An Investment Problem / 507
18.7 Production and Sales Scheduling / 511
18.8 Summary / 515 Problems / 516

19. Classical Optimization Techniques / 523
19.1 Basic Concepts / 524
19.1A—Gradient Vectors / 524
19.1B—The Hessian Matrix / 525
19.1C—Determinants / 526
19.1D—Quadratic Forms / 529
19.1E—Concave and Convex Functions / 530
19.2 Unconstrained Multiple-Variable Problems / 534
19.2A—A Concave Objective Function / 534
19.2B—The General Objective Function / 535
19.3 Constrained Problems and the Kuhn-Tucker Conditions / 536
19.4 Summary / 542 Problems / 542

20. Nonlinear Programming / 547
20.1 Quadratic Programming / 548
20.1A—The Kuhn-Tucker Conditions for the QP Problem / 548
20.1B—Solving the QP Problem / 551
20.1C—An Example: Selecting a Portfolio / 556
20.2 Separable Programming / 558
20.3 Separable Convex Programming / 563
20.4 The Gradient-Search Method / 568
20.5 Summary / 570 Problems / 571

PART THREE STOCHASTIC MODELS / 577

21. Introduction to Queuing Theory (Poisson, Single-Server, and FCFS Models) / 579
21.1 Introduction to Queuing Theory / 580
21.2 Definitions and Notation / 581
21.3 The Pure-Birth and Pure-Death Models / 583
21.4 Steady-State Results / 588
21.5 Relationships Among L, L_q, W, and W_q / 591
21.6 The (M/M/1):(∞/∞/FCFS) System / 591
21.7 The (M/M/1):(N/∞/FCFS) System / 596
21.8 The (M/M/1):(N/N/FCFS) System / 599
21.9 Summary / 602 Problems / 603

22. Queuing Models: The Multiple-Server Model and Other Variations / 609
22.1 The (M/M/c):(∞/∞/FCFS) System / 609
22.2 The (M/M/c):(∞/∞/SIRO) System / 613
22.3 The (M/G/1):(∞/∞/G) System / 615
22.4 The (M/D/1):(∞/∞/G) System / 618
22.5 The ($M/E_k/1$):(∞/∞/G) System / 619
22.6 The ($M_i/M_i/c$):(∞/∞/NPPR) System / 621
22.7 Summary and Survey of Other Systems / 627 Problems / 631

23. Queuing Models as a Tool in Decision Making / 639
23.1 *The Optimal μ in the (M/M/1):(∞/∞/FCFS) Model; the Cost of μ Linear and μ Continuous / 640*
23.2 *The Optimal μ in the (M/M/1):(∞/∞/FCFS) Model; μ Discrete / 642*
23.3 *The Optimal μ in the (M/E_k/1):(∞/∞/G) Model; the Cost of μ Nonlinear / 645*
23.4 *The Optimal μ and N in the (M/M/1):(N/∞/G) Model; Both μ and N Discrete / 647*
23.5 *The Optimal c in the (M/M/c):(∞/∞/G) Model; the Cost of c Linear / 648*
23.6 *The Optimal c, μ, and λ in the (M/M/c):(∞/∞/G) Model / 650*
23.7 *The Optimal c and μ in the (M/M/c):(∞/∞/G) Model; μ Discrete and the Cost of μ a Special Nonlinear Case / 652*
23.8 *Aspiration Models / 653*
23.9 *Summary / 654* *Problems / 655*

24. Introduction to Stochastic Inventory Models / 665
24.1 *The Single-Order Model / 666*
24.1A—Discrete Demand and No Ordering Costs / 666
24.1B—Continuous Demand and No Ordering Costs / 671
24.1C—Beginning Inventory and Order Costs Included / 672
24.2 *A Continuous-Review Model with Discrete Demand / 675*
24.3 *A Continuous-Review Model with Continuous Demand / 683*
24.4 *A Periodic-Review Model with Continuous Demand / 685*
24.5 *Survey of Extensions in Inventory Modeling / 691*
24.6 *Summary / 693* *Problems / 694*

25. Simulation / 701
25.1 *Overview of Simulation / 702*
25.2 *Generating Random Numbers / 704*
25.3 *Generating Random Variables from a Probability Distribution / 708*
25.3A—The Uniform Distribution / 708
25.3B—The Exponential Distribution / 709
25.3C—The Normal Distribution / 710
25.3D—The Poisson Distribution / 711
25.3E—An Empirical Discrete Distribution / 712
25.3F—Additional Considerations / 713
25.4 *Keeping Track of Time / 713*
25.5 *Variance-Reduction Techniques / 718*
25.6 *Problems of Designing and Implementing a Simulation Study / 720*
25.7 *Special Computer Simulation Languages / 721*
25.8 *Summary / 721* *Problems / 722*

APPENDIX A COMPACT NOTATION / 727

APPENDIX B REVIEW OF MATRIX ALGEBRA / 729
B.1 Definitions / 729
B.2 Algebra / 730
B.2A—Addition and Subtraction / 730
B.2B—Multiplication by a Scalar / 730
B.2C—Multiplication of Matrices / 731
B.2D—Special Kinds of Matrices / 733
B.3 Vectors / 733
B.4 Matrix Inverse and Uses / 734
B.4A—Finding the Inverse / 734
B.4B—Solving Linear Systems / 737
B.5 Matrix Operations Involving the Transpose / 738
Problems / 739

APPENDIX C CALCULUS REVIEW / 741
C.1 The Limit / 741
C.2 The Derivative / 743
C.3 Optimization Techniques / 744
C.4 Integration / 745
C.5 Area Under the Curve / 746
C.6 Derivatives and Integrals of Functions with Several Variables / 747
C.7 Differentiating Under the Integral / 748
Problems / 749

APPENDIX D PROBABILITY THEORY / 752
D.1 Basic Definitions / 752
D.2 Probability and Density Functions / 757
D.3 Expectation and Variance / 761
D.4 Some Useful Discrete Probability Functions / 764
D.5 Some Useful Continuous Density Functions / 767
D.6 Summary of Distributions and Selection of the Appropriate Function / 769
D.7 Joint Probability Distributions / 771
Problems / 776

APPENDIX E TABLES / 783

INDEX / 787