

Contents

1	Introduction	1
1.1	Dirichlet Characters	1
1.2	Modular Forms I	8
1.3	Modular Forms II.....	21
1.4	Paramodular Forms I.....	32
1.5	Paramodular Forms II	45
1.6	Local Results	52
1.7	Results About Siegel Modular Forms	59
1.8	Further Directions	66
 Part I Local Theory		
2	Background	71
2.1	Some Definitions	71
2.2	Representations.....	74
2.3	The Paramodular Theory.....	86
3	Stable Klingen Vectors	91
3.1	The Stable Klingen Subgroup	91
3.2	Stable Klingen Vectors	94
3.3	Paramodularization	95
3.4	Operators on Stable Klingen Vectors	96
3.5	Level Raising Operators	97
3.6	Four Conditions	103
3.7	Level Lowering Operators	106
3.8	Stable Hecke Operators	110
3.9	Commutation Relations	116
3.10	A Result About Eigenvalues	121
4	Some Induced Representations	123
4.1	Double Coset Representatives	123
4.2	Stable Klingen Vectors in Siegel Induced Representations	127

4.3	Non-Existence of Certain Vectors	137
4.4	Characterization of Paramodular Vectors	141
5	Dimensions	147
5.1	The Upper Bound	147
5.2	The Shadow of a Newform	151
5.3	Zeta Integrals and Diagonal Evaluation	154
5.4	Dimensions for Some Generic Representations	156
5.5	Dimensions for Some Non-Generic Representations.....	158
5.6	The Table of Dimensions	162
5.7	Some Consequences.....	163
6	Hecke Eigenvalues and Minimal Levels	167
6.1	At the Minimal Stable Klingen Level	167
6.2	Non-Generic Paramodular Representations.....	172
6.3	At the Minimal Paramodular Level.....	173
6.4	An Upper Block Algorithm	179
7	The Paramodular Subspace	181
7.1	Calculation of Certain Zeta Integrals	181
7.2	Generic Representations	185
7.3	Non-Generic Representations.....	190
7.4	Summary Statements	191
8	Further Results About Generic Representations	193
8.1	Non-Vanishing on the Diagonal	194
8.2	The Kernel of a Level Lowering Operator	196
8.3	The Alternative Model	205
8.4	A Lower Bound on the Paramodular Level	210
9	Iwahori-Spherical Representations	221
9.1	Some Background.....	221
9.2	Action of the Iwahori-Hecke Algebra	226
9.3	Stable Hecke Operators and the Iwahori-Hecke Algebra	231
9.4	Characteristic Polynomials	234
 Part II Siegel Modular Forms		
10	Background on Siegel Modular Forms	245
10.1	Basic Definitions	245
10.2	Modular Forms	247
11	Operators on Siegel Modular Forms	259
11.1	Overview.....	259
11.2	Level Raising Operators	261
11.3	A Level Lowering Operator.....	268
11.4	Hecke Operators.....	273
11.5	Some Relations Between Operators	286

12	Hecke Eigenvalues and Fourier Coefficients	287
12.1	Applications	288
12.2	Another Formulation	300
12.3	Examples	311
12.4	Computing Eigenvalues	322
12.5	A Recurrence Relation	325
A	Tables	335
	References	351
	Index	357