

## CONTENTS

<i>Introduction to the series</i>	<b>v</b>
<i>Preface</i>	<b>ix</b>
<b>1. INTRODUCTION</b>	<b>1</b>
<b>1.1. A bit of history</b>	<b>2</b>
<b>1.2. Purposes of book</b>	<b>4</b>
<b>1.3. Organization of the book</b>	<b>5</b>
<b>References</b>	<b>6</b>
<b>Part I. Mathematical programming and competitive equilibria</b>	<b>9</b>
<b>2. MATHEMATICAL PROGRAMMING AND SPACELESS AND SPATIAL EQUILIBRIUM</b>	<b>11</b>
<b>2.1. General mathematical programming model</b>	<b>11</b>
(a) The Kuhn-Tucker conditions	13
(b) The dual non-linear programming problem	16
(c) Solution algorithms	17
<b>2.2. Quadratic programming model</b>	18
(a) The Kuhn-Tucker conditions	19
(b) The dual quadratic programming problem	20
(c) Reducibility theorem	20
(d) Solution algorithms	20
<b>2.3. Linear programming model</b>	21
(a) The Kuhn-Tucker conditions	22
(b) Solution algorithms	23
<b>2.4. Competitive economic equilibrium, Pareto optimality and programming models</b>	23
<b>2.4.1. Competitive equilibrium</b>	24
(a) Economic environment	24
(b) Competitive spaceless equilibrium	25
(c) General competitive spatial equilibrium	26
(d) Partial competitive spatial equilibrium.	30
(e) Pareto optimum	31

2.4.2.	Spatial price equilibrium . . . . .	33
(a)	Net quasi-welfare approach . . . . .	35
(b)	Maximum net revenue approach . . . . .	38
2.4.3.	Concluding remarks . . . . .	40
	Notes and references . . . . .	41
 <i>Part II. Linear pricing and allocation models</i> . . . . .		45
3.	LINEAR DISTRIBUTION MODELS . . . . .	47
3.1.	Economic environment . . . . .	47
3.2.	The mathematical model . . . . .	48
(a)	The optimality conditions . . . . .	49
(b)	Economic interpretation . . . . .	50
(c)	Dual interpretation . . . . .	51
(d)	The programming tableau and solution algorithm . . . . .	53
(e)	An example . . . . .	54
3.3.	Extensions of the model . . . . .	58
(a)	Total regional supplies unequal to total regional demands . . . . .	58
(b)	Production and transport cost minimization models . . . . .	59
(c)	Net revenue maximization model . . . . .	60
(d)	Multiple commodity transportation model . . . . .	62
	Notes and references . . . . .	63
4.	SIMPLE LINEAR MODELS OF PRODUCTION AND ALLOCATION . . . . .	65
4.1.	Economic environment . . . . .	65
4.2.	Mathematical model . . . . .	66
(a)	Notation . . . . .	66
(b)	The programming model . . . . .	67
(c)	The optimality conditions . . . . .	68
(d)	Economic interpretation . . . . .	70
(e)	Dual interpretation . . . . .	72
(f)	Tableau and solution algorithm . . . . .	73
(g)	An example . . . . .	73
4.3.	A revenue maximization formulation . . . . .	76
(a)	Economic environment . . . . .	78
(b)	The programming model . . . . .	78
(c)	The optimality conditions . . . . .	79
(d)	Economic interpretation . . . . .	80
4.4.	Extensions of the model . . . . .	81
(a)	A multi-product formulation . . . . .	81
4.5.	Summary . . . . .	84
	Notes and references . . . . .	85
5.	AN INTERREGIONAL LINEAR ACTIVITY ANALYSIS MODEL . . . . .	86
5.1.	Economic environment . . . . .	86

5.2.	Mathematical model . . . . .	88
(a)	Notation . . . . .	88
(b)	The programming model . . . . .	89
(c)	The optimality conditions . . . . .	91
(d)	Economic interpretation . . . . .	93
(e)	Dual formulation . . . . .	96
(f)	The programming tableau . . . . .	97
(g)	An illustrative example . . . . .	97
(h)	A numerical example . . . . .	100
5.3.	Discussion of the model . . . . .	102
	Notes and references . . . . .	103

*Part III. Single product spatial price equilibrium models . . . . .* 105

6. GENERAL SPATIAL PRICE EQUILIBRIUM MODELS . . . . . 107

6.1.	Single product case . . . . .	107
(a)	Economic environment . . . . .	107
(b)	Mathematical programming model . . . . .	110
(c)	Optimality conditions and their economic interpretation . . . . .	110
(d)	Diagrammatical explanation . . . . .	112
6.2.	Multi-commodity case (integrable case) . . . . .	113
(a)	Economic environment . . . . .	113
(b)	Mathematical programming model . . . . .	117
(c)	Optimality conditions and their economic interpretation . . . . .	120
6.3.	Multi-commodity case (non-integrable case) . . . . .	121
(a)	Economic environment . . . . .	121
(b)	Mathematical model . . . . .	121
(c)	Optimality conditions and their properties . . . . .	122
6.4.	Concluding remarks . . . . .	126
	Notes and references . . . . .	127

7. SINGLE PRODUCT SPATIAL PRICE EQUILIBRIUM MODEL – QUANTITY FORMULATION 129

7.1.	Economic environment and notation . . . . .	129
7.2.	The mathematical model . . . . .	131
(a)	The programming formulation (quantity) . . . . .	131
(b)	The optimality conditions . . . . .	133
(c)	Economic interpretation . . . . .	134
(d)	Programming aspects of the model and solution algorithm . . . . .	137
(e)	Existence and uniqueness of solution . . . . .	142
(f)	Illustrative numerical example . . . . .	142
7.3.	Demands and/or supplies fixed in some regions . . . . .	145
(a)	Economic environment and notation . . . . .	145
(b)	Mathematical programming formulation and programming tableau . . . . .	145
(c)	Illustrative numerical example . . . . .	148
7.4.	Concluding remarks . . . . .	150
	Notes and references . . . . .	150

8.	SINGLE PRODUCT SPATIAL PRICE EQUILIBRIUM MODEL – PRICE FORMULATION . . . . .	153
8.1.	Economic environment and notation . . . . .	153
8.2.	The dual of the quantity formulation (problem 7.1) . . . . .	154
8.3.	Mathematical model – price formulation . . . . .	157
(a)	Programming formulation . . . . .	157
(b)	The optimality conditions and economic interpretation . . . . .	158
(c)	Programming aspects of the model . . . . .	160
(d)	Existence and uniqueness of a solution . . . . .	164
(e)	Illustrative numerical example . . . . .	164
8.4.	Demands and supplies fixed in some regions . . . . .	166
(a)	Mathematical programming formulation and programming tableau . . . . .	166
(b)	Illustrative numerical example . . . . .	169
	Notes and references . . . . .	172
9.	MODIFIED SPECIFICATIONS OF THE SINGLE PRODUCT SPATIAL PRICE EQUILIBRIUM MODEL . . . . .	173
9.1.	Mathematical model (quantity) . . . . .	173
(a)	The quantity formulation . . . . .	173
(b)	Economic interpretation of the model . . . . .	174
(c)	Optimality conditions and their economic interpretation . . . . .	182
(d)	Existence and uniqueness of a solution, and programming aspects of the model . . . . .	184
9.2.	Mathematical model (price) . . . . .	184
(a)	Price formulation . . . . .	184
(b)	Economic interpretation of the model . . . . .	186
(c)	Optimality conditions and the economic interpretation . . . . .	192
(d)	Existence, uniqueness of a solution and programming tableau . . . . .	193
9.3.	Modified mathematical models . . . . .	193
(a)	Simplified excess supply model . . . . .	194
9.4.	Concluding remarks . . . . .	195
	Notes and references . . . . .	195
10.	SOME EXTENSIONS OF THE PRICE EQUILIBRIUM MODELS . . . . .	197
10.1.	Import tariff and/or export subsidy . . . . .	197
(a)	Definition . . . . .	197
(b)	Mathematical programming model . . . . .	199
(c)	Illustrative numerical example . . . . .	201
10.2.	Import-export quota . . . . .	202
(a)	Definition . . . . .	202
(b)	Mathematical programming model . . . . .	204
(c)	Illustrative numerical example . . . . .	205
	Notes and references . . . . .	207
11.	SINGLE PRODUCT MONOPOLY SPATIAL EQUILIBRIUM MODELS . . . . .	208
11.1.	The spatial monopoly problem . . . . .	208
(a)	Economic environment . . . . .	208

(b) Definition of monopolistic spatial equilibrium . . . . .	210
(c) The programming formulation . . . . .	211
(d) The optimality conditions and their interpretation . . . . .	211
(e) The dual and programming tableau . . . . .	213
(f) Illustrative numerical example . . . . .	214
11.2. Precautionary monopolist . . . . .	217
(a) Mathematical model . . . . .	218
(b) The programming problem . . . . .	220
(c) The optimality conditions . . . . .	220
(d) The dual and programming tableau . . . . .	222
(e) A numerical example . . . . .	222
11.3. Benevolent monopolist with arbitragers . . . . .	225
(a) Definition . . . . .	225
(b) Mathematical model . . . . .	226
(c) A numerical example . . . . .	227
11.4. Concluding remarks . . . . .	230
Notes and references . . . . .	230

*Part IV. Multi-product spatial price equilibrium models . . . . .* 233

12. MULTI-PRODUCT SPATIAL EQUILIBRIUM MODELS . . . . .	235
12.1. Economic environment and notation . . . . .	235
12.2. Mathematical model . . . . .	237
(a) The programming formulation . . . . .	237
(b) The optimality conditions . . . . .	240
(c) Economic interpretation . . . . .	240
(d) Solution algorithm . . . . .	242
(e) Existence and uniqueness . . . . .	244
12.3. Mathematical model – price formulation . . . . .	244
12.4. A numerical example . . . . .	247
12.5. Mathematical model – asymmetric quadratic matrix . . . . .	248
12.6. Concluding remarks . . . . .	256
Notes and references . . . . .	257
13. MULTI-PRODUCT SPATIAL EQUILIBRIUM MODELS: SOME EXTENSIONS . . . . .	258
13.1. Import tariff and/or export subsidy . . . . .	258
(a) Definition . . . . .	258
(b) Mathematical programming model . . . . .	260
13.2. Fixed demand and/or supply and import quota . . . . .	261
(a) Definition . . . . .	261
(b) Mathematical programming model . . . . .	263
13.3. Ad valorem tariffs . . . . .	264
(a) Definition . . . . .	264
(b) Mathematical programming model . . . . .	265
13.4. Other possible modifications and extensions . . . . .	266
13.5. Illustrative numerical examples . . . . .	267
(a) Basic model . . . . .	267

(b) Import tariff case . . . . .	268
(c) Import quota case . . . . .	269
(d) Ad valorem tariff case . . . . .	272
(e) Linear regional demand and/or supply functions for both final and intermediate or primary products . . . . .	272
Notes and references . . . . .	274
<b>14. SPATIAL PRICE EQUILIBRIUM ACTIVITY ANALYSIS MODEL . . . . .</b>	<b>275</b>
14.1. Economic environment . . . . .	275
14.2. Mathematical model (quantity formulation) . . . . .	278
(a) The programming formulation . . . . .	278
(b) The optimality conditions . . . . .	280
(c) Economic interpretation . . . . .	281
(d) Primal-dual formulation . . . . .	282
(e) Programming solution existence . . . . .	286
(f) Programming tableau and solution algorithm . . . . .	287
(g) Illustrative numerical example . . . . .	287
14.3. Mathematical model (price formulation) . . . . .	289
14.4. Asymmetric $\Omega$ or $B$ ( $A$ or $\Sigma$ ) . . . . .	292
14.5. Discussion of the model . . . . .	295
Notes and references . . . . .	297
<b>Part V. General competitive spaceless and spatial equilibrium models</b>	<b>299</b>
<b>15. COMPETITIVE GENERAL EQUILIBRIUM MODEL FOR A CLOSED ECONOMY . . . . .</b>	<b>301</b>
15.1. Economic environment . . . . .	301
(a) Consumers . . . . .	301
(b) Producers . . . . .	302
(c) Economic system . . . . .	303
(d) Behavioral assumptions . . . . .	303
(e) Conditions for general equilibrium under perfect competition . . . . .	304
15.2. Mathematical programming approach . . . . .	305
15.3. Existence proof . . . . .	310
(a) Case I . . . . .	310
(b) Case II . . . . .	312
15.4. Concluding remarks . . . . .	316
Notes and references . . . . .	317
<b>16. GENERAL COMPETITIVE EQUILIBRIUM IN INTERNATIONAL AND INTERREGIONAL TRADE . . . . .</b>	<b>318</b>
16.1. Economic environment, notation and definition . . . . .	318
16.2. Definition . . . . .	322
16.3. Existence of a competitive international trade equilibrium . . . . .	323
(a) Mathematical programming approach . . . . .	323
(b) The fixed point mapping and the existence proof . . . . .	326
16.4. Concluding remarks . . . . .	329
References . . . . .	329

<i>Part VI. Pricing and allocation over time . . . . .</i>	331
17. INTERTEMPORAL PRICE EQUILIBRIUM MODELS . . . . .	333
17.1. A single product intertemporal price equilibrium model . . . . .	333
(a) Economic environment . . . . .	333
(b) Notation and definitions . . . . .	333
(c) Mathematical programming model . . . . .	335
(d) Equilibrium conditions . . . . .	338
(e) Programming tableau, existence and uniqueness of a solution . . . . .	342
(f) Price formulation . . . . .	343
(g) Illustrative and numerical examples . . . . .	345
17.2. Multi-commodity intertemporal price equilibrium . . . . .	352
17.2.1. Linear demand and supply function model . . . . .	352
(a) Definition and notation . . . . .	352
(b) Mathematical programming model . . . . .	353
(c) The equilibrium conditions . . . . .	357
(d) Existence and uniqueness of a solution and programming tableau . . . . .	358
(e) Price formulations and some modifications . . . . .	359
(f) Some remarks . . . . .	360
17.2.2. Linear demand function and activity analysis supply model . . . . .	360
(a) Definition and notation . . . . .	361
(b) Mathematical programming model . . . . .	363
(c) The equilibrium conditions . . . . .	365
(d) Existence of a solution and programming tableau . . . . .	367
17.2.3. Illustrative numerical example . . . . .	367
17.3. Concluding remarks . . . . .	372
References . . . . .	373
18. INTERTEMPORAL-SPATIAL PRICE EQUILIBRIUM MODELS . . . . .	374
18.1. A multi-product finite horizon intertemporal spatial equilibrium . . . . .	374
(a) Economic environment . . . . .	374
(b) Definition of intertemporal spatial equilibrium . . . . .	377
18.2. Mathematical formulation (integrable case) . . . . .	379
(a) Quadratic programming model . . . . .	379
(b) Optimality conditions and intertemporal spatial equilibrium conditions . . . . .	384
(c) The dual problem . . . . .	387
(d) Programming tableau . . . . .	389
18.3. Modifications and extensions . . . . .	389
(a) A multi-period storage spatial equilibrium model . . . . .	389
18.4. Mathematical formulation (non-integrable case) . . . . .	398
18.5. Multi-region one-storage period intertemporal activity analysis models . . . . .	399
(a) Economic environment . . . . .	399
(b) Mathematical programming formulation . . . . .	401
(c) The optimality and intertemporal spatial equilibrium conditions . . . . .	403
(d) Asymmetric demand coefficient case (non-integrable case) . . . . .	407
18.6. Numerical example . . . . .	407
(a) Spatial and intertemporal equilibrium in water pricing and allocation . . . . .	407
Notes and references . . . . .	411

19.	ADAPTIVE INTERTEMPORAL SPATIAL EQUILIBRIUM MODELS . . . . .	412
19.1.	Economic environment . . . . .	412
19.2.	Mathematical formulation . . . . .	413
(a)	Linear demand and supply functions (integrable case) . . . . .	413
(b)	Linear demand and supply functions (non-integrable case) . . . . .	415
(c)	Linear demand function and activity analysis supply model (integrable case) . . . . .	415
(d)	Linear demand function and activity analysis supply model (non-integrable case) . . . . .	416
(e)	Extensions, modifications and duals . . . . .	416
19.3.	Illustrative and numerical examples . . . . .	416
19.4.	Concluding remarks . . . . .	423
	References . . . . .	426
20.	CONCLUDING REMARKS . . . . .	427
	References . . . . .	431
	Appendix A. Mathematical foundation . . . . .	433
	Appendix B. Saddle point and saddle value problem . . . . .	454
	Appendix C. Gradient method . . . . .	457
	Appendix D. Gradient methods for solving non-linear programming problems . .	460
	Appendix E. The Kuhn-Tucker equivalence theorem . . . . .	463
	Appendix F. Simplex procedure applied to a quadratic programming problem . .	468
	Appendix G. Stability and comparative static properties of a quadratic spatial equilibrium . . . . .	484
	Appendix H. Alternative solution algorithms for spatial equilibrium problems .	490
	Appendix I. Duality theorems in linear and non-linear programming . . . . .	499
	Appendix J. Some properties of the Hitchcock-Koopmans transportation model .	504
	Appendix K. The Samuelsonian intertemporal price equilibrium model . . . .	506
	Appendix L. Linear non-normative models of trade . . . . .	515
	<i>Author index</i> . . . . .	522
	<i>Subject index</i> . . . . .	525