
Contents

Preface	xiii
1 Introduction	1
1.1 Purpose	1
1.2 Background	2
1.2.1 The EM algorithm	3
1.2.2 Markov chain Monte Carlo	3
1.3 Why analysis by simulation?	4
1.4 Looking ahead	6
1.4.1 Scope of the rest of this book	6
1.4.2 Knowledge assumed on the part of the reader	7
1.4.3 Software and computational details	7
1.5 Bibliographic notes	8
2 Assumptions	9
2.1 The complete-data model	9
2.2 Ignorability	10
2.2.1 Missing at random	10
2.2.2 Distinctness of parameters	11
2.3 The observed-data likelihood and posterior	11
2.3.1 Observed-data likelihood	11
2.3.2 Examples	13
2.3.3 Observed-data posterior	17
2.4 Examining the ignorability assumption	20
2.4.1 Examples where ignorability is known to hold	20
2.4.2 Examples where ignorability is not known to hold	22
2.4.3 Ignorability is relative	23
2.5 General ignorable procedures	23
2.5.1 A simulated example	24

2.5.2	Departures from ignorability	26
2.5.3	Notes on nonignorable alternatives	28
2.6	The role of the complete-data model	29
2.6.1	Departures from the data model	29
2.6.2	Inference treating certain variables as fixed	31
3	EM and data augmentation	37
3.1	Introduction	37
3.2	The EM algorithm	37
3.2.1	Definition	37
3.2.2	Examples	41
3.2.3	EM for posterior modes	46
3.2.4	Restrictions on the parameter space	46
3.2.5	The ECM algorithm	49
3.3	Properties of EM	51
3.3.1	Stationary values	51
3.3.2	Rate of convergence	55
3.3.3	Example	59
3.3.4	Further comments on convergence	61
3.4	Markov chain Monte Carlo	68
3.4.1	Gibbs sampling	69
3.4.2	Data augmentation	70
3.4.3	Examples of data augmentation	73
3.4.4	The Metropolis-Hastings algorithm	78
3.4.5	Generalizations and hybrid algorithms	79
3.5	Properties of Markov chain Monte Carlo	80
3.5.1	The meaning of convergence	80
3.5.2	Examples of nonconvergence	80
3.5.3	Rates of convergence	83
4	Inference by data augmentation	89
4.1	Introduction	89
4.2	Parameter simulation	90
4.2.1	Dependent samples	90
4.2.2	Summarizing a dependent sample	93
4.2.3	Rao-Blackwellized estimates	98
4.3	Multiple imputation	104
4.3.1	Bayesianly proper multiple imputations	105
4.3.2	Inference for a scalar quantity	107
4.3.3	Inference for multidimensional estimands	112
4.4	Assessing convergence	118
4.4.1	Monitoring convergence in a single chain	119

4.4.2	Monitoring convergence with parallel chains	126
4.4.3	Choosing scalar functions of the parameter	128
4.4.4	Convergence of posterior summaries	131
4.5	Practical guidelines	134
4.5.1	Choosing a method of inference	135
4.5.2	Implementing a parameter-simulation experiment	136
4.5.3	Generating multiple imputations	138
4.5.4	Choosing an imputation model	139
4.5.5	Further comments on imputation modeling	143
5	Methods for normal data	147
5.1	Introduction	147
5.2	Relevant properties of the complete-data model	148
5.2.1	Basic notation	148
5.2.2	Bayesian inference under a conjugate prior	150
5.2.3	Choosing the prior hyperparameters	154
5.2.4	Alternative parameterizations and sweep	157
5.3	The EM algorithm	163
5.3.1	Preliminary manipulations	163
5.3.2	The E-step	164
5.3.3	Implementation of the algorithm	166
5.3.4	EM for posterior modes	170
5.3.5	Calculating the observed-data loglikelihood	173
5.3.6	Example: serum-cholesterol levels of heart-attack patients	175
5.3.7	Example: changes in heart rate due to marijuana use	178
5.4	Data augmentation	181
5.4.1	The I-step	181
5.4.2	The P-step	183
5.4.3	Example: cholesterol levels of heart-attack patients	185
5.4.4	Example: changes in heart rate due to marijuana use	189
6	More on the normal model	193
6.1	Introduction	193
6.2	Multiple imputation: example 1	193
6.2.1	Cholesterol levels of heart-attack patients	193
6.2.2	Generating the imputations	194
6.2.3	Complete-data point and variance estimates	194

9.3.2 Likelihood inference for restricted models	344
9.3.3 Bayesian inference	346
9.4 Algorithms for incomplete mixed data	348
9.4.1 Predictive distributions	348
9.4.2 EM for the unrestricted model	352
9.4.3 Data augmentation	355
9.4.4 Algorithms for restricted models	357
9.5 Data examples	359
9.5.1 St. Louis Risk Research Project	359
9.5.2 Foreign Language Attitude Scale	367
9.5.3 National Health and Nutrition Examination Survey	372
10 Further topics	379
10.1 Introduction	379
10.2 Extensions of the normal model	379
10.2.1 Restricted covariance structures	379
10.2.2 Heavy-tailed distributions	380
10.2.3 Interactions	380
10.2.4 Semicontinuous variables	381
10.3 Random-effects models	382
10.4 Models for complex survey data	383
10.5 Nonignorable methods	384
10.6 Mixture models and latent variables	384
10.7 Coarsened data and outlier models	385
10.8 Diagnostics	386
Appendices	
A Data examples	387
B Storage of categorical data	395
C Software	399
References	401
Index	415

6.2.4	Combining the estimates	197
6.2.5	Alternative choices for the number of imputations	197
6.3	Multiple imputation: example 2	200
6.3.1	Predicting achievement in foreign language study	200
6.3.2	Applying the normal model	202
6.3.3	Exploring the observed-data likelihood and posterior	204
6.3.4	Overcoming the problem of inestimability	206
6.3.5	Analysis by multiple imputation	208
6.4	A simulation study	211
6.4.1	Simulation procedures	212
6.4.2	Complete-data inferences	214
6.4.3	Results	216
6.5	Fast algorithms based on factored likelihoods	218
6.5.1	Monotone missingness patterns	218
6.5.2	Computing alternative parameterizations	220
6.5.3	Noniterative inference for monotone data	223
6.5.4	Monotone data augmentation	226
6.5.5	Implementation of the algorithm	229
6.5.6	Uses and extensions	234
6.5.7	Example	236
7	Methods for categorical data	239
7.1	Introduction	239
7.2	The multinomial model and Dirichlet prior	240
7.2.1	The multinomial distribution	240
7.2.2	Collapsing and partitioning the multinomial	243
7.2.3	The Dirichlet distribution	247
7.2.4	Bayesian inference	250
7.2.5	Choosing the prior hyperparameters	251
7.2.6	Collapsing and partitioning the Dirichlet	255
7.3	Basic algorithms for the saturated model	257
7.3.1	Characterizing an incomplete categorical dataset	257
7.3.2	The EM algorithm	260
7.3.3	Data augmentation	264
7.3.4	Example: victimization status from the National Crime Survey	267
7.3.5	Example: Protective Services Project for Older Persons	272

7.4	Fast algorithms for near-monotone patterns	275
7.4.1	Factoring the likelihood and prior density	275
7.4.2	Monotone data augmentation	279
7.4.3	Example: driver injury and seatbelt use	282
8	Loglinear models	289
8.1	Introduction	289
8.2	Overview of loglinear models	289
8.2.1	Definition	289
8.2.2	Eliminating associations	292
8.2.3	Sufficient statistics	294
8.2.4	Model interpretation	295
8.3	Likelihood-based inference with complete data	297
8.3.1	Maximum-likelihood estimation	297
8.3.2	Iterative proportional fitting	298
8.3.3	Hypothesis testing and goodness of fit	302
8.3.4	Example: misclassification of seatbelt use and injury	303
8.4	Bayesian inference with complete data	305
8.4.1	Prior distributions for loglinear models	305
8.4.2	Inference using posterior modes	307
8.4.3	Inference by Bayesian IPF	308
8.4.4	Why Bayesian IPF works	312
8.4.5	Example: misclassification of seatbelt use and injury	318
8.5	Loglinear modeling with incomplete data	320
8.5.1	ML estimates and posterior modes	320
8.5.2	Goodness-of-fit statistics	322
8.5.3	Data augmentation and Bayesian IPF	324
8.6	Examples	325
8.6.1	Protective Services Project for Older Persons	325
8.6.2	Driver injury and seatbelt use	328
9	Methods for mixed data	333
9.1	Introduction	333
9.2	The general location model	334
9.2.1	Definition	334
9.2.2	Complete-data likelihood	336
9.2.3	Example	338
9.2.4	Complete-data Bayesian inference	339
9.3	Restricted models	341
9.3.1	Reducing the number of parameters	341