

Contents

Preface	ix
Conventions on Terminology	xii
1. Sets	
1.1 Finite, Countable and Uncountable Sets	1
1.2 Zorn's Lemma and Well-ordered Sets	8
1.3 Graphs	15
2. Groups	
2.1 Definition and Basic Properties	25
2.2 Permutation Groups	32
2.3 The Isomorphism Theorems	34
2.4 Soluble and Nilpotent Groups	37
2.5 Commutators	42
2.6 The Frattini Subgroup and the Fitting Subgroup	46
3. Lattices and Categories	
3.1 Definitions; Modular and Distributive Lattices	51
3.2 Chain Conditions	60
3.3 Categories	65
3.4 Boolean Algebras	70
4. Rings and Modules	
4.1 The Definitions Recalled	79
4.2 The Category of Modules over a Ring	84
4.3 Semisimple Modules	91
4.4 Matrix Rings	96
4.5 Direct Products of Rings	101
4.6 Free Modules	105
4.7 Projective and Injective Modules	110
4.8 The Tensor Product of Modules	117
4.9 Duality of Finite Abelian Groups	125

5. Algebras		
5.1	Algebras; Definition and Examples	131
5.2	The Wedderburn Structure Theorems.....	137
5.3	The Radical	141
5.4	The Tensor Product of Algebras	146
5.5	The Regular Representation; Norm and Trace.....	153
5.6	Möbius Functions	157
6. Multilinear Algebra		
6.1	Graded Algebras	165
6.2	Free Algebras and Tensor Algebras	168
6.3	The Hilbert Series of a Graded Ring or Module.....	173
6.4	The Exterior Algebra on a Module.....	179
7. Field Theory		
7.1	Fields and their Extensions	189
7.2	Splitting Fields.....	195
7.3	The Algebraic Closure of a Field.....	200
7.4	Separability.....	203
7.5	Automorphisms of Field Extensions.....	206
7.6	The Fundamental Theorem of Galois Theory	211
7.7	Roots of Unity.....	217
7.8	Finite Fields.....	223
7.9	Primitive Elements; Norm and Trace	227
7.10	Galois Theory of Equations	232
7.11	The Solution of Equations by Radicals	238
8. Quadratic Forms and Ordered Fields		
8.1	Inner Product Spaces.....	249
8.2	Orthogonal Sums and Diagonalization	252
8.3	The Orthogonal Group of a Space.....	256
8.4	The Clifford Algebra and the Spinor Norm	259
8.5	Witt's Cancellation Theorem and the Witt Group of a Field	268
8.6	Ordered Fields	272
8.7	The Field of Real Numbers.....	275
8.8	Formally Real Fields.....	279
8.9	The Witt Ring of a Field.....	291
8.10	The Symplectic Group	298
8.11	Quadratic Forms in Characteristic Two	301
9. Valuation Theory		
9.1	Divisibility and Valuations.....	307
9.2	Absolute Values.....	312
9.3	The p -adic Numbers	322
9.4	Integral Elements.....	331
9.5	Extension of Valuations	336

10. Commutative Rings	
10.1 Operations on Ideals.....	347
10.2 Prime Ideals and Factorization.....	349
10.3 Localization	354
10.4 Noetherian Rings.....	361
10.5 Dedekind Domains	362
10.6 Modules over Dedekind Domains	371
10.7 Algebraic Equations	376
10.8 The Primary Decomposition	380
10.9 Dimension.....	386
10.10 The Hilbert Nullstellensatz.....	391
11. Infinite Field Extensions	
11.1 Abstract Dependence Relations	397
11.2 Algebraic Dependence.....	402
11.3 Simple Transcendental Extensions	405
11.4 Separable and p -radical Extensions	409
11.5 Derivations.....	414
11.6 Linearly Disjoint Extensions	418
11.7 Composites of Fields.....	427
11.8 Infinite Algebraic Extensions	431
11.9 Galois Descent	437
11.10 Kummer Extensions.....	441
Bibliography	449
List of Notations	453
Author Index	457
Subject Index	459