Contents

Pr	eface		xiii	
	Note	es to Preface	xvi	
	Refe	xvi		
1	Introduction, basic concepts and reminders			
	1.1	What is a financial security?	1	
	1.2	The financial market	2	
	1.3	Choices made by investors	4	
	1.4	Equilibrium	5	
	1.5	Basic reminders on probability and statistics	7	
	1.6	Reminders on optimization	12	
	1.7	Conclusion	15	
	Notes to Chapter 1		16	
	Refe	rences	18	

vi Financial Securities

Par	t I	Equilibriu	m in the stock market	19
2	The		f a rational individual towards risk	21
2	2.1	The thec	ory of von Neumann, Morgenstern and Savage	22
	2.2	Risk ave	rsion in the sense of Arrow and Pratt (1964–1965)	30
	2.3		t of utility functions	33
	2.4		levelopments	37
	2.5	_		41
	Apr	endix 2.1	 Outline proof of the expected utility theorem 	42
		es to Chap		44
		erences		48
•	Do.	rtfolio choi	ines	51
3	3.1		cal example with two financial securities	52
	3.1	Rick rec	duction: the effect of correlation between securities	54
	3.2		st-order condition for portfolio choice in	
	3.5		eral case of n securities	57
		3.3.1	The rate of return on a portfolio of n securities	57
		3.3.2		58
		3.3.3	a to a large of divor	
			risk aversion	61
	3.4	The eff	cient frontier and separation theorems	63
		3.4.1	The Markowitz frontier	63
		3.4.2	The Black separation theorem	64
		3.4.3.	The Tobin separation theorem	65
	3.	5 Implen	nentation	66
	3.6 Conclusion		68	
	Aj	ppendix 3.1	1 – Properties of equation system [3.6a,c]	40
			and the Black theorem	69
	\mathbf{A}_{1}	ppendix 3.2	2 – Introduction to some Bayesian considerations	70
	N	otes to Cha	apter 3	73
	R	eferences		74
4	ı T	he Capital	Asset Pricing Model: statement and use	77
		.1 The C	APM	78
		4.1.1	Definition of the market portfolio	78
		4.1.2	Proof and statement of the CAPM	78

		4.1.3	Another statement of the CAPM: measurement	
			of risk by the beta coefficient	80
		4.1.4	The calculation of beta and one more	
			interpretation of the CAPM	81
	4.2	The m	acro-economic context of the CAPM	84
		4.2.1	The case of an infinitely elastic supply of securities	84
		4.2.2	The case of an infinitely inelastic supply of securities	s 85
	4.3	Identif	ying over- or under-valued securities	87
	4.4	Measu	ring the performance of portfolio managers	93
	4.5	Choice	e of investment projects	97
	4.6	Recent	t developments	101
	4.7	Conclu	usion	105
	Арре	endix – J	Justification of Jensen's $lpha$	106
	Note	s to Cha	ipter 4	107
	Refe	rences		110
5	Criti	que and	l evaluation of the CAPM	113
	5.1	-	atic nature of the CAPM	114
		5.1.1	Single-period versus multi-period models	114
		5.1.2	IID versus non-IID rates of return:	
			information or instrumental variables	115
		5.1.3	Conditional and unconditional CAPM	117
	5.2	Conce	ptual shortcomings of the CAPM	118
		5.2.1	Roll's critique	118
		5.2.2	Hansen and Richard's critique	119
		5.2.3	A digression on performance measurement	120
	5.3	Stabili	ity of the components of the CAPM	121
		5.3.1	Stability of the betas	121
		5.3.2	Stability of expected returns for securities and	
			the market; stability of the slope of the market line	124
		5.3.3	Instrumental variables used to specify the	
			conditional CAPM	127
	5.4	Empir	rical tests of the CAPM	128
		5.4.1	Early studies: the work of	
			[Fama and MacBeth 1973]	129
		5.4.2	The multivariate method illustrated by the	
			work of [Gibbons 1982] and [Gibbons et al. 1989]	130

viii Financial Securities

		5.4.3	The GMM illustrated by the work of [Harvey 1989]	132
		5.4.4	Other statistical models	136
	5.5	Anoma	lies	139
	5.6	Conclu	sion	141
	Note	s to Cha	pter 5	143
Pai	rt II (Options	and arbitrage	151
	Note	s to the	introduction to Part II	154
	Refe	rences		155
6	Opti	on prici	ng by the arbitrage method	157
	6.1	Option	s: definitions and payoffs	158
	6.2	Princip	les of option pricing: discrete-time	161
		6.2.1	Options with one period to maturity	161
		6.2.2	'Risk-neutral' probabilities	164
		6.2.3	Options with any number of periods to maturity:	
			the binomial technique	166
	6.3	Contin	nuous time: the Black-Scholes formula	168
	6.4	Impler	nentation	172
		6.4.1	The stochastic process of the underlying asset	172
		6.4.2	The constant volatility hypothesis	173
		6.4.3	The constant interest rate assumption	175
		6.4.4	Exercise before maturity and dividends paid	
			on the underlying asset	177
		6.4.5	Estimation of the volatility parameter s	179
	6.5	Empir	rical tests	180
	6.6 Other secondary securities and other			
		applie	ations of the arbitrage principle	182
		6.6.1	Futures contracts on financial securities	183
		6.6.2	Index options	184
		6.6.3	Options on futures contracts written on	
			financial securities	185
		6.6.4	Currency options	186
		6.6.5	Exchange options	187
		6.6.6	'Primes'	187
		6.6.7	Portfolio insurance	188
	6.7	Concl	lusion	189
	App	endix		190

			Content	ts ix			
	Note	s to Cha	pter 6	193			
	Refe	rences		197			
7	Opti	on prici	ng in continuous time	205			
	7.1	Basic co	oncepts of stochastic differential calculus	206			
		7.1.1	Brownian motion	206			
		7.1.2	Brownian motion as a limit of the random walk	207			
		7.1.3	Stochastic differential equations, Itō processes				
			and diffusion processes	210			
		7.1.4	Ito's lemma or change of variable formula	214			
	7.2	Option	pricing in continuous time	215			
		7.2.1	The Black-Scholes partial differential equation	217			
		7.2.2	Solution of the Black-Scholes partial				
			differential equation	220			
	7.3	Options	s on securities paying a continuous dividend	221			
	7.4	Americ	ean options	222			
		7.4.1	Options with discrete and certain dividend	222			
		7.4.2	Boundary conditions applicable to American				
			options on securities paying a continuous				
			dividend: 'smooth-pasting condition'	224			
		7.4.3	Approximate solution methods	226			
	7.5	Introd	uction to numerical methods	229			
		7.5.1	The finite difference method	230			
		7.5.2	Another numerical method: Monte Carlo simulation	233			
	Con	clusion		234			
	App	endix 1 -	- Variations of Brownian motion	236			
	Appendix 2 – The stochastic integral with respect to						
			Brownian motion	239			
	Note	es to Cha	inter 7	242			
		rences	.,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	245			
8	Eval	uation o	of the liabilities and assets of a limited				
	company; financial engineering 2						
	8.1		ation of ordinary debt	250			
		8.1.1	Single, zero-coupon debt with a one-off payment	250			
		8.1.2	Payments made to shareholders during				
			the lifetime of the debt	253			
		813	Payments made to debtholders	255			

x Financial Securities

		8.1.4	Subordinated debt	251
	8.2	Costs o	of the various sources of funds	260
	8.3	Evalua	tion of convertible bonds	264
		8.3.1	A textbook case	264
		8.3.2	Dividends paid during the lifetime of the bond	266
		8.3.3	Callable convertible debt	268
		8.3.5	Implementation of the binomial technique	269
		8.3.6	Warrants and exercise strategy	271
	8.4	Evalua	tion of assets	273
		8.4.1	Evaluation of investment projects	274
		8.4.2	Calculation of expanded net present value	276
		8.4.3	Determining the optimum date at which to invest	278
	Con	elusion		279
	Appe	endix – C	Optimization of the value of a mine	280
	Note	s to Cha	pter 8	282
	Refe	rences		285
Pa	rt III	Bonds a	nd the term structure of interest rates	289
9	The	term str	ucture of interest rates	291
	9.1	Remine	der on discounting	292
	9.2	Yield to	o maturity, duration and convexity	293
		9.2.1	Yield to maturity	293
		9.2.2	Duration	297
		9.2.3	The link between yield to maturity and duration	299
		9.2.4	Convexity	302
	9.3	Measu	ring the term structure of interest rates	304
	9.4	The in	fluence of taxes	306
	9.5	Immur	nization	307
	9.6	Traditi	onal theories of the term structure of interest rates	311
		9.6.1	The expectations hypothesis	311
		9.6.2	Liquidity preference and preferred habitat	315
	Cone	elusion		316
	Appe	endix 1 -	- Bond repurchases (sinking funds)	317
	Appe	endix 2 -	The expectations hypothesis in continuous time	321
	Note	s to Cha	pter 9	323
	Refe	rences		327

10	The r	novemei	nt of interest rates and the price of bonds	331		
	10.1	The pri	ce of a bond on a market subject to			
		one ran	dom factor	332		
	10.2	Exampl	e: the Vasicek formula	336		
	10.3	The price of a bond on a market subject to				
		two ran	dom factors	340		
	10.4	The Co.	x, Ingersoll and Ross model	344		
	10.5	New for	rms of bond	346		
	10.6	Recent developments				
	Notes	Notes to Chapter 10				
	References			352		
11	Options on interest rates and on interest-sensitive instruments			35 3		
	11.1	Remind	ler on 'risk-neutral' probabilities	354		
	11.2	The Bla	ack, Derman and Toy example	356		
	11.3	Specifying the behaviour of the term structure				
		in continuous time				
		11.3.1	Specifying the behaviour of bond prices	359		
		11.3.2	Specifying the behaviour of forward interest			
			rates in continuous time	360		
		11.3.3	Specifying the behaviour of spot rates in			
			continuous time	361		
	11.4	Pricing	options in the Black, Derman and Toy example	362		
	11.5	Pricing options in continuous time				
	Conclusion			366		
	Notes to Chapter 11			367		
	Refe	rences		370		
Ind	lex			373		