

Contents

Preface	xi
Chapter 1. The Life and Work of É. Cartan	1
§1.1. Parents' home	1
§1.2. Student at a school and a lycée	2
§1.3. University student	4
§1.4. Doctor of Science	6
§1.5. Professor	8
§1.6. Academician	17
§1.7. The Cartan family	24
§1.8. Cartan and the mathematicians of the world	27
Chapter 2. Lie Groups and Algebras	33
§2.1. Groups	33
§2.2. Lie groups and Lie algebras	37
§2.3. Killing's paper	42
§2.4. Cartan's thesis	45
§2.5. Roots of the classical simple Lie groups	46
§2.6. Isomorphisms of complex simple Lie groups	51
§2.7. Roots of exceptional complex simple Lie groups	51
§2.8. The Cartan matrices	53
§2.9. The Weyl groups	55
§2.10. The Weyl affine groups	60
§2.11. Associative and alternative algebras	63
§2.12. Cartan's works on algebras	67
§2.13. Linear representations of simple Lie groups	69
§2.14. Real simple Lie groups	73
§2.15. Isomorphisms of real simple Lie groups	78
§2.16. Reductive and quasireductive Lie groups	82
§2.17. Simple Chevalley groups	84
§2.18. Quasigroups and loops	85

Chapter 3. Projective Spaces and Projective Metrics	87
§3.1. Real spaces	87
§3.2. Complex spaces	93
§3.3. Quaternion spaces	95
§3.4. Octave planes	96
§3.5. Degenerate geometries	97
§3.6. Equivalent geometries	101
§3.7. Multidimensional generalizations of the Hesse transfer principle	107
§3.8. Fundamental elements	109
§3.9. The duality and triality principles	113
§3.10. Spaces over algebras with zero divisors	116
§3.11. Spaces over tensor products of algebras	118
§3.12. Degenerate geometries over algebras	121
§3.13. Finite geometries	123
Chapter 4. Lie Pseudogroups and Pfaffian Equations	125
§4.1. Lie pseudogroups	125
§4.2. The Kac-Moody algebras	127
§4.3. Pfaffian equations	129
§4.4. Completely integrable Pfaffian systems	130
§4.5. Pfaffian systems in involution	132
§4.6. The algebra of exterior forms	134
§4.7. Application of the theory of systems in involution	135
§4.8. Multiple integrals, integral invariants, and integral geometry	136
§4.9. Differential forms and the Betti numbers	139
§4.10. New methods in the theory of partial differential equations	142
Chapter 5. The Method of Moving Frames and Differential Geometry	145
§5.1. Moving trihedra of Frénet and Darboux	145
§5.2. Moving tetrahedra and pentaspheres of Demoulin	147
§5.3. Cartan's moving frames	148
§5.4. The derivational formulas	150
§5.5. The structure equations	152
§5.6. Applications of the method of moving frames	153
§5.7. Some geometric examples	154
§5.8. Multidimensional manifolds in Euclidean space	158
§5.9. Minimal manifolds	160
§5.10. "Isotropic surfaces"	162
§5.11. Deformation and projective theory of multidimensional manifolds	166

§5.12. Invariant normalization of manifolds	170
§5.13. "Pseudo-conformal geometry of hypersurfaces"	174
Chapter 6. Riemannian Manifolds. Symmetric Spaces	177
§6.1. Riemannian manifolds	177
§6.2. Pseudo-Riemannian manifolds	181
§6.3. Parallel displacement of vectors	181
§6.4. Riemannian geometry in an orthogonal frame	183
§6.5. The problem of embedding a Riemannian manifold into a Euclidean space	184
§6.6. Riemannian manifolds satisfying "the axiom of plane"	185
§6.7. Symmetric Riemannian spaces	186
§6.8. Hermitian spaces as symmetric spaces	191
§6.9. Elements of symmetry	193
§6.10. The isotropy groups and orbits	196
§6.11. Absolutes of symmetric spaces	198
§6.12. Geometry of the Cartan subgroups	199
§6.13. The Cartan submanifolds of symmetric spaces	200
§6.14. Antipodal manifolds of symmetric spaces	201
§6.15. Orthogonal systems of functions on symmetric spaces	202
§6.16. Unitary representations of noncompact Lie groups	204
§6.17. The topology of symmetric spaces	207
§6.18. Homological algebra	209
Chapter 7. Generalized Spaces	211
§7.1. "Affine connections" and Weyl's "metric manifolds"	211
§7.2. Spaces with affine connection	212
§7.3. Spaces with a Euclidean, isotropic, and metric connection	215
§7.4. Affine connections in Lie groups and symmetric spaces with an affine connection	216
§7.5. Spaces with a projective connection	219
§7.6. Spaces with a conformal connection	220
§7.7. Spaces with a symplectic connection	221
§7.8. The relativity theory and the unified field theory	222
§7.9. Finsler spaces	223
§7.10. Metric spaces based on the notion of area	225
§7.11. Generalized spaces over algebras	226
§7.12. The equivalence problem and G -structures	228
§7.13. Multidimensional webs	231
Conclusion	235
Dates of Cartan's Life and Activities	239
List of Publications of Élie Cartan	241

Appendix A.	Rapport sur les Travaux de M. Cartan, by H. Poincaré	263
Appendix B.	Sur une dégénérescence de la géométrie euclidienne, by É. Cartan	273
Appendix C.	Allocution de M. Élie Cartan	275
Appendix D.	The Influence of France in the Development of Mathematics	281
Bibliography		303