CONTENTS

	Chapter I	. Sets	and	ele	men	tary	set	O	per	ati	ons						
. Sets					٠.												
. Elements	of a set																
. Symbols	ϵ and ϵ																
	sting of one ele																
	y set																
-	of sets																
. Sets of se	ots																
. Subset of	a set																
. Sum of s	ets																
. Difference	of sets																
. Product of	of sets																
. Disjoint s	ums																
. Compleme	ent of a set .																
-	oairs																
	dence. Function																
-	e corresponden																
	product of set																
	iation of sets																
. Finite an . Fundame: . Effectivel . Various t	at sets. Relation d infinite sets atal properties y equivalent se heorems on the or-Bernstein Tl	of the ets	· · · · rela · · · · valen	tion	of s	ets						 	•		· · ·		
. Effective	Chapter II able and effecti denumerability denumerability denumerability	vely of the of the	lenur ie sei he ir	nera t of ifini	ble all	set rat	s . iona of r	l n lon	un -01	abe	ers lap	 pin	g	int	er	va	ls

488 Contents

6.	Non-denumerable sets	47
7.	Properties of sets containing denumerable subsets	49
8	Sets infinite in the sense of Dedekind	
Q.	Various definitions of finite sets	53
10	Denumerability of the Cartesian product of two denumerable sets	54 50
10.	Designorability of the Cartesian product of two denumerable sets	56
	Chapter IV. Sets of the power of the continuum	
1.	Sets of the power of the continuum and sets effectively of the power of the	
1.	continuum	= 7
9	Non-denumerability of the set of real numbers	57 = 7
	Removing a denumerable set from a set of the power of the continuum	5 7
1	Set of real numbers of an arbitrary interval	58
7. 5	Sum of two sets of the power of the continuum	60
e.	Cartesian product of a denumerable set and a set of the power of the con-	62
υ.	tinuum	00
7	Set of all infinite sequences of natural numbers	63
g.	Cartesian product of two sets of the power of the continuum	63
Q.	Impossibility of a continuous 1-1 mapping of a plane on a straight line	65
10	Continuous curve filling up a square	70
11	Set of all infinite sequences of real numbers	71
11.	Continuous curve filling up a denumerably dimensional cube	73 77
12.	Set of all continuous functions	79
	Decomposition of a set of natural numbers into a continuum of almost	19
17.	disjoint sets	81
	Chapter V. Comparing the power of sets	
1.	Sets of different power	84
2.	Sets of greater power than finite sets and denumerable sets. Hypothesis	01
	of the continuum	86
3.	Cantor's theorem on the set of all subsets of a given set	87
4.	Generalized Continuum Hypothesis	88
5.	Forming sets of ever greater powers	89
	G seems because it is a seem of the seems of	00
	Chapter VI. Axiom of choice	
1.	The axiom of choice. Controversy about it	92
2.	The axiom of choice for a finite set of sets	96
	The axiom of choice for an infinite sequence of sets	97
	Hilbert's axiom	97
	General principle of choice	98
5.	Axiom of choice for finite sets	101
	Examples of cases where we are able and where we are not able to	
	make an effective choice	109
7.	Applications of the axiom of choice	112
	The m-to-n correspondence	129
	Dependent choices	132

Can	tan	+

	Contents
	Chapter VII. Cardinal numbers and operations on them
1.	Cardinal numbers
2.	Sum of cardinal numbers
3.	Product of two cardinal numbers
	Exponentiation of cardinal numbers
5.	Power of the set of all subsets of a given set
	Chapter VIII. Inequalities for cardinal numbers
1	Definition of an inequality between two cardinal numbers
1. 0	Transitivity of the relation of inequality Addition of inequalities
۷. 9	Exponentiation of inequalities for cardinal numbers
	Relation $\mathfrak{m} \leqslant *\mathfrak{n}$
	Cl IV Difference of condition numbers
	Chapter IX. Difference of cardinal numbers
1.	Theorem of A. Tarski and F. Bernstein
2.	Theorem on increasing the diminuend
3.	Theorem on increasing the subtrahend
4.	Difference in which the subtrahend is a natural number
5.	Proof of the formula $2^{m} - m = 2^{m}$ for $m > \aleph_{0}$ without the aid of the axiom
	of choice
6.	Quotient of cardinal numbers
	Chapter X. Infinite series and infinite products of cardinal numbers
1.	Sum of an infinite series of cardinal numbers
	Properties of an infinite series of cardinal numbers
3.	Examples of infinite series of cardinal numbers
4.	Sum of an arbitrary set of cardinal numbers
5.	Infinite product of cardinal numbers
6.	Properties of infinite products of cardinal numbers. Examples
	Theorem of J. König
8.	Product of an arbitrary set of cardinal numbers
	Chapter XI. Ordered sets
1.	Ordered sets
2.	Partially ordered sets
3.	Lattices
4.	Similarity of sets
5 .	First and last element of an ordered set. Cuts. Jumps. Density and con-
	tinuity of a set
6.	Finite ordered sets
7.	Sets of type ω
8.	Sets of type n
9.	Dense ordered sets as subsets of continuous sets
10.	Sets of type λ

490 Contents

Chapter XII. Order types and operations on them

1.	Order types	225
2.	Sum of two order types	226
3.	Product of two order types	232
4.	Sum of an infinite series of order types	239
5.	Power of the set of all denumerable order types	242
	Power of the set of all order types of the power of the continuum	243
	Sum of an arbitrary ordered set of order types	246
8.	Infinite products of order types	247
9.	Segments and remainders of order types	250
10.	Divisors of order types	254
11	Comparison of order types	257
	comparison of order types	201
	Chapter XIII. Well-ordered sets	
1	Well-ordered sets	261
	The principle of transfinite induction	262
	Induction for ordered sets	264
	Similar mapping of a well-ordered set on its subset	267
Ŧ.	Proportion of comments of real ordered set of Drivered there are real	207
θ.	Properties of segments of well-ordered sets. Principal theorem on well-ordered sets	0.00
	ordered sets	268
	Chapter XIV. Ordinal numbers	
1.	Ordinal numbers. Ordinal numbers as indices of the elements of well-	
	ordered sets	272
2.	Sets of ordinal numbers	273
3.	Sum of ordinal numbers	274
4.	Properties of the sum of ordinal numbers. Numbers of the 1-st and of	214
-•	the 2-nd kind	277
5	Remainders of ordinal numbers	280
6.	Prime components	282
7	Transfinite sequences of ordinal numbers and their limits	287
8	Infinite series of ordinal numbers and their sums	290
a.	Product of ordinal numbers	293
10	Properties of the product of ordinal numbers	295 295
11	Theorem on the division of ordinal numbers	297
19	Divisors of ordinal numbers	301
	Prime factors of ordinal numbers	306
14.	Certain properties of prime components	308
	Exponentiation of ordinal numbers	309
16.	Definitions by transfinite induction	314
17.	Transfinite products of ordinal numbers	
18.	Properties of the powers of ordinal numbers	316
19.		318
20.	The power ω^a . Normal expansions of ordinal numbers	318 322
	Epsilon numbers	318 322 226
21.	Epsilon numbers	318 322
21. 22.	Epsilon numbers	318 322 226
21. 22. 23.	Epsilon numbers	318 322 326 330

	491
25. On ordinal numbers commutative with respect to addition	346 351 362 366 367
Chapter XV. Number classes and alephs	
 Numbers of the 1-st and of the 2-nd class Cardinal number %1	369 372 378 382 389 390 391
 8. Formula \(\mathbb{N}_a^2 = \mathbb{N}_a\) and conclusions from it	400
sive alephs	405
Chapter XVI. Zermelo's theorem and other theorems equivalent to the axiom of cl	hoic
1. Equivalence of the axiom of choice, of Zermelo's theorem and of the pro-	
blem of trichatomy	
	410
2. Various theorems on cardinal numbers equivalent to the axiom of choice	41'
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	41 429
2. Various theorems on cardinal numbers equivalent to the axiom of choice	41° 429 43°
 Various theorems on cardinal numbers equivalent to the axiom of choice A. Lindenbaum's theorem	41 429 43
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	41 429 43 43
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	411 429 430 43 44 44 44 45
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	411 429 430 43 44 44 44 45 45
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	41 42 43 43 44 44 45 45 45
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	411 429 430 43 44 44 44 45
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	411 429 439 43 44 44 45 45 45
2. Various theorems on cardinal numbers equivalent to the axiom of choice 3. A. Lindenbaum's theorem	41 42 43 43 44 44 45 46