

Content

1 Neighborhood Structures

1.1 Finite Graphs

1.1.1 Historical Remarks	1
1.1.2 Elementary Theory of Sets and Relations	3
1.1.3 Elementary Graph Theory	4

1.2 Neighborhood Graphs

1.2.1 Graph Theory and Image Processing	9
1.2.2 Points, Edges, Paths, and Regions	12
1.2.3 Matrices of Adjacency	15
1.2.4 Graph Distances	17

1.3 Components in Neighborhood Structures

1.3.1 Search in Graphs and Labyrinths	19
1.3.2 Neighborhood Search	20
1.3.3 Graph Search in Images	22
1.3.4 Neighbored Sets and Separated Sets	25
1.3.5 Component Labeling	27

1.4 Dilatation and Erosion

1.4.1 Metric Spaces	30
1.4.2 Boundaries and Cores in Neighborhood Structures	32
1.4.3 Set Operations and Set Operators	35
1.4.4 Dilatation and Erosion	36
1.4.5 Opening and Closing	38

2 Incidence Structures

2.1 Homogeneous Incidence Structures

2.1.1 Topological Problems	42
2.1.2 Cellular Complexes	44
2.1.3 Incidence Structures	48
2.1.4 Homogeneous Incidence Structures	50
2.1.5 Z^n as Incidence Structure	53

2.2 Oriented Neighborhood Structures

2.2.1 Orientation of a Neighborhood Structure	57
2.2.2 Euler Characteristic of a Neighborhood Structure	59
2.2.3 Border Meshes and Separation Theorem	63
2.2.4 Search in Oriented Neighborhood Structures	66
2.2.5 Coloring in Oriented Neighborhood Structures	68

2.3 Homogeneous Oriented Neighborhood Structures

2.3.1 Homogeneity in Neighborhood Structures	72
2.3.2 Toroidal Nets	73
2.3.3 Curvature of Border Meshes in Toroidal Nets	76
2.3.4 Planar Semi-Homogeneous Graphs	78

2.4 Objects in N-Dimensional Incidence Structures

2.4.1 Three-Dimensional Homogeneous Incidence Structures	82
2.4.2 Objects in Z^n	85
2.4.3 Similarity of Objects	89
2.4.4 General Surface Formulas	91
2.4.5 Interpretation of Object Characteristics	94

3 Topological Laws and Properties**3.1 Objects and Surfaces**

3.1.1 Surfaces in Discrete Spaces	99
3.1.2 Contur Following as Two-Dimensional Boundary Detection . .	100
3.1.3 Three-Dimensional Surface Detection	102
3.1.4 Curvature of Conturs and Surfaces	105

3.2 Motions and Intersections

3.2.1 Motions of Objects in Z^n	109
3.2.2 Count Measures and Intersections of Objects	111
3.2.3 Applications of Intersection Formula	113
3.2.4 Count Formulas	116
3.2.5 Stochastic Images	120

3.3 Topology Preserving Operations

3.3.1 Topological Equivalence	125
3.3.2 Simple Points	127
3.3.3 Thinning	131

4 Geometrical Laws and Properties

4.1 Discrete Geometry

4.1.1	Geometry and Number Theory	133
4.1.2	Minkowski Geometry	136
4.1.3	Translative Neighborhood Structures	139
4.1.4	Digitalization Effects	143

4.2 Straight Lines

4.2.1	Rational Geometry	148
4.2.2	Digital Straight Lines in Z^2	151
4.2.3	Continued Fractions	153
4.2.4	Straight Lines in Z^n	157

4.3 Convexity

4.3.1	Convexity in Discrete Geometry	161
4.3.2	Maximal Convex Objects	163
4.3.3	Determination of Convex Hull	169
4.3.4	Convexity in Z^n	171

4.4 Approximative Motions

4.4.1	Pythagorean Rotations	173
4.4.2	Shear Transformations	175
4.4.3	General Affine Transformations	178

5 Discrete Functions

5.1 One-Dimensional Periodical Discrete Functions

5.1.1	Functions	179
5.1.2	Space of Periodical Discrete Function	181
5.1.3	LSI-Operators and Convolutions	183
5.1.4	Products of Linear Operators	186

5.2 Algebraic Theory of Discrete Functions

5.2.1	Domain of Definition and Range of Values	188
5.2.2	Algebraical Structures	190
5.2.3	Convolution of Functions	195
5.2.4	Convolution Orthogonality	196

5.3 Orthogonal Convolution Bases

5.3.1	General Properties in OCB's	199
5.3.2	Fourier Transform	201

5.3.3 Number Theoretical Transforms	203
5.3.4 Two-Dimensional NTT	208
5.4 Inversion of Convolutions	
5.4.1 Conditions for Inverse Elements	213
5.4.2 Deconvolutions and Texture Synthesis	216
5.4.3 Approximative Computation of Inverse Elements	218
5.4.4 Theory of Approximative Inversion	220
5.4.5 Examples of Inverse Filters	221
5.5 Differences and Sums of Functions	
5.5.1 Differences of One-Dimensional Discrete Functions	225
5.5.2 Difference Equations and Z-Transform	227
5.5.3 Sums of Functions	228
5.5.4 Bernoulli's Polynomials	230
5.5.5 Determination of Moments	232
5.5.6 Final Comments	236
6 Summary and Symbols	237
7 References	248
8 Index	265