

Contents

Introduction

1 EQUATIONAL VARIETIES	1
1.1 ALGEBRAS ON AN OPERATOR DOMAIN	1
1.2 FREE ALGEBRAS AND THE TERMS	3
1.3 EQUATIONAL VARIETIES	5
2 CANONICAL FORMS	11
2.1 SOLVING EQUATIONS	11
2.2 REWRITING SYSTEMS AND THE COMPLETION ALGORITHM	15
2.2.1 <i>Rewriting systems</i>	15
2.2.2 <i>The completion algorithm</i>	20
2.3 THE ASSOCIATIVE COMMUTATIVE CASE	27
2.3.1 <i>Equivalence, matching and unification</i>	28
2.3.2 <i>Rewriting systems of associative-commutative theories</i>	36
2.4 REWRITING ORDERINGS AND CANONICAL SYSTEMS	41
2.4.1 <i>Rewriting orderings</i>	41
2.4.2 <i>Canonical systems</i>	44
3 MONOIDS and GROUPS	53
3.1 SEMIGROUPS AND MONOIDS	54
3.1.1 <i>Semigroups</i>	54
3.1.2 <i>Some results on Thue systems</i>	56
3.1.3 <i>Termination</i>	58
3.2 GROUPS	60
3.2.1 <i>Completion</i>	60
3.2.2 <i>A detailed example</i>	65
3 ABELIAN CASE	68
3.3.1 <i>Semigroups and monoids</i>	68
3.3.2 <i>Groups</i>	71
4 RINGS, MODULES and ALGEBRAS	81
4.1 RINGS	81
4.1.1 <i>Non-abelian rings</i>	81
4.1.2 <i>Abelian rings</i>	87
4.1.3 <i>Distributive lattices</i>	88
4.3 MODULES AND ALGEBRAS	90
4.3.1 <i>Modules</i>	91
4.3.2 <i>Algebras</i>	94
4.5 SOME EXAMPLES	96

5 DEHN ALGORITHM and the SYMMETRIZATION	101
5.1 DEFINITIONS	102
5.2 THE SYMMETRIZATION	104
5.3 CRITICAL PAIR REDUCTION BY CONTEXTS	110
5.3.1 <i>Complete Absorption of β or γ by F-reduction</i>	111
5.3.2 <i>Left context reduction</i>	112
5.3.3 <i>Right context reduction</i>	117
5.4 CONFLUENCE OF DEHN ALGORITHM	119
5.5 EXAMPLES	132
 6 COMPLETE GROUP PRESENTATIONS	 139
6.1 THE SEVENTEEN PLANE GROUPS	140
6.2 SURFACE GROUPS	150
6.2.1 <i>Orientable surfaces</i>	150
6.2.2 <i>Non-orientable surfaces</i>	155
6.3 COXETER GROUPS	158
6.3.1 <i>Definition of Coxeter groups</i>	158
6.3.2 <i>Completion of a Coxeter group</i>	159
6.3.3 <i>A Reduction algorithm</i>	168
6.3.4 <i>Some examples</i>	171
6.4 POLYHEDRAL GROUPS	173
6.4.1 <i>Polyhedral groups with odd number of generators</i>	178
6.4.2 <i>Polyhedral groups with even number of generators</i>	180
6.5 SYMMETRIC GROUPS	182
6.5.1 <i>Presentation with adjacent transpositions</i>	182
6.5.2 <i>Presentation with all transpositions</i>	183
 References	 187