

Contents

Preface	vii
----------------	-----

Chapter 0

Preliminaries	1
----------------------	---

Part 1: Preliminaries. Matrices. Determinants. Polynomials. Functions. Equivalence Relations. Zorn's Lemma. Cardinality.
Part 2: Algebraic Structures. Groups. Rings. Integral Domains. Ideals and Principal Ideal Domains. Prime Elements. Fields. The Characteristic of a Ring.

Part 1 Basic Linear Algebra

Chapter 1

Vector Spaces	27
----------------------	----

Vector Spaces. Subspaces. The Lattice of Subspaces. Direct Sums. Spanning Sets and Linear Independence. The Dimension of a Vector Space. The Row and Column Space of a Matrix. Coordinate Matrices. Exercises.

Chapter 2

Linear Transformations	45
-------------------------------	----

Linear Transformations. The Kernel and Image of a Linear Transformation. Isomorphisms. The Rank Plus Nullity Theorem. Linear Transformations from F^n to F^m . Change of Basis Matrices. The Matrix of a Linear Transformation. Change of Bases for Linear Transformations. Equivalence of Matrices. Similarity of Matrices. Invariant Subspaces and Reducing Pairs. Exercises.

Chapter 3

The Isomorphism Theorems

63

Quotient Spaces. The First Isomorphism Theorem. The Dimension of a Quotient Space. Additional Isomorphism Theorems. Linear Functionals. Dual Bases. Reflexivity. Annihilators. Operator Adjoints. Exercises.

Chapter 4

Modules I

83

Motivation. Modules. Submodules. Direct Sums. Spanning Sets. Linear Independence. Homomorphisms. Free Modules. Summary. Exercises.

Chapter 5

Modules II

97

Quotient Modules. Quotient Rings and Maximal Ideals. Noetherian Modules. The Hilbert Basis Theorem. Exercises.

Chapter 6

Modules over Principal Ideal Domains

107

Free Modules over a Principal Ideal Domain. Torsion Modules. The Primary Decomposition Theorem. The Cyclic Decomposition Theorem for Primary Modules. Uniqueness. The Cyclic Decomposition Theorem. Exercises.

Chapter 7

The Structure of a Linear Operator

121

A Brief Review. The Module Associated with a Linear Operator. Submodules and Invariant Subspaces. Orders and the Minimal Polynomial. Cyclic Submodules and Cyclic Subspaces. Summary. The Decomposition of V . The Rational Canonical Form. Exercises.

Chapter 8

Eigenvalues and Eigenvectors

135

The Characteristic Polynomial of an Operator. Eigenvalues and Eigenvectors. The Cayley-Hamilton Theorem. The Jordan Canonical Form. Geometric and Algebraic Multiplicities. Diagonalizable Operators. Projections. The Algebra of Projections. Resolutions of the Identity. Projections and Diagonalizability. Projections and Invariance. Exercises.

Chapter 9

Real and Complex Inner Product Spaces

157

Introduction. Norm and Distance. Isometries. Orthogonality. Orthogonal and Orthonormal Sets. The Projection Theorem. The Gram-Schmidt Orthogonalization Process. The Riesz Representation Theorem. Exercises.

Chapter 10

The Spectral Theorem for Normal Operators

175

The Adjoint of a Linear Operator. Orthogonal Diagonalizability. Motivation. Self-Adjoint Operators. Unitary Operators. Normal Operators. Orthogonal Diagonalization. Orthogonal Projections. Orthogonal Resolutions of the Identity. The Spectral Theorem. Functional Calculus. Positive Operators. The Polar Decomposition of an Operator. Exercises.

Part 2 Topics

Chapter 11

Metric Vector Spaces

205

Symmetric, Skew-symmetric and Alternate Forms. The Matrix of a Bilinear Form. Quadratic Forms. Linear Functionals. Orthogonality. Orthogonal Complements. Orthogonal Direct Sums. Quotient Spaces. Symplectic Geometry—Hyperbolic Planes. Orthogonal Geometry—Orthogonal Bases. The Structure of an Orthogonal Geometry. Isometries. Symmetries. Witt's Cancellation Theorem. Witt's Extension Theorem. Maximum Hyperbolic Subspaces. Exercises.

Chapter 12

Metric Spaces

239

The Definition. Open and Closed Sets. Convergence in a Metric Space. The Closure of a Set. Dense Subsets. Continuity. Completeness. Isometries. The Completion of a Metric Space. Exercises.

Chapter 13

Hilbert Spaces

263

A Brief Review. Hilbert Spaces. Infinite Series. An Approximation Problem. Hilbert Bases. Fourier Expansions. A Characterization of Hilbert Bases. Hilbert Dimension. A Characterization of Hilbert Spaces. The Riesz Representation Theorem. Exercises.

Chapter 14

Tensor Products

291

Free Vector Spaces. Another Look at the Direct Sum. Bilinear Maps and Tensor Products. Properties of the Tensor Product. The Tensor Product of Linear Transformations. Change of Base Field. Multilinear Maps and Iterated Tensor Products. Alternating Maps and Exterior Products. Exercises.

Chapter 15

Affine Geometry

315

Affine Geometry. Affine Combinations. Affine Hulls. The Lattice of Flats. Affine Independence. Affine Transformations. Projective Geometry. Exercises.

Chapter 16

The Umbral Calculus

329

Formal Power Series. The Umbral Algebra. Formal Power Series as Linear Operators. Sheffer Sequences. Examples of Sheffer Sequences. Umbral Operators and Umbral Shifts. Continuous Operators on the Umbral Algebra. Operator Adjoints. Automorphisms of the Umbral Algebra. Derivations of the Umbral Algebra. Exercises.

References

353

Index of Notation

355

Index

357