

## Table of contents

Chapter I - REVIEW OF MULTIPLICITY THEORY .....	1
§ 1 The multiplicity symbol .....	1
§ 2 Hilbert functions .....	6
§ 3 Generalized multiplicities and Hilbert functions .....	10
§ 4 Reductions and integral closure of ideals .....	16
§ 5 Faithfully flat extensions .....	25
§ 6 Projection formula and criterion for multiplicity one .....	27
§ 7 Examples .....	34
Chapter II - $\mathbb{Z}$ -GRADED RINGS AND MODULES .....	44
§ 8 Associated graded rings and Rees algebras .....	44
§ 9 Dimension .....	49
§ 10 Homogeneous parameters .....	55
§ 11 Regular sequences on graded modules .....	68
§ 12 Review on blowing up .....	77
§ 13 Standard bases .....	88
§ 14 Examples .....	100
Appendix - Homogeneous subrings of a homogeneous ring .....	112
Chapter III - ASYMPTOTIC SEQUENCES AND QUASI-UNMIXED RINGS .....	117
§ 15 Auxiliary results on integral dependence of ideals ...	117
§ 16 Associated primes of the integral closure of powers of an ideal .....	122
§ 17 Asymptotic sequences .....	133
§ 18 Quasi-unmixed rings .....	137
§ 19 The theorem of Rees-Böger .....	146
Chapter IV - VARIOUS NOTIONS OF EQUIMULTIPLE AND PERMISSIBLE IDEALS .....	152
§ 20 Reinterpretation of the theorem of Rees-Böger .....	152
§ 21 Hironaka-Grothendieck homomorphism .....	159
§ 22 Projective normal flatness and numerical characterization of permissibility .....	166
§ 23 Hierarchy of equimultiplicity and permissibility .....	182
§ 24 Open conditions and transitivity properties .....	194

Chapter V - EQUIMULTPLICITY AND COHEN-MACAULAY PROPERTY OF BLOWING UP RINGS .....	204
§25 Graded Cohen-Macaulay rings .....	205
§26 The case of hypersurfaces .....	212
§27 Transitivity of Cohen-Macaulayness of Rees rings .....	223
Appendix (K. Yamagishi and U. Orbanz) - Homogeneous domains of minimal multiplicity .....	230
Chapter VI - CERTAIN INEQUALITIES AND EQUALITIES OF HILBERT FUNCTIONS AND MULTIPLICITIES .....	240
§28 Hyperplane sections .....	240
§29 Quadratic transformations .....	243
§30 Semicontinuity .....	250
§31 Permissibility and blowing up of ideals .....	253
§32 Transversal ideals and flat families .....	258
Chapter VII - LOCAL COHOMOLOGY AND DUALITY OF GRADED RINGS .....	270
§33 Review on graded modules .....	270
§34 Matlis duality .....	289
Part I : Local case .....	289
Part II: Graded case .....	293
§35 Local cohomology .....	295
§36 Local duality for graded rings .....	310
Appendix - Characterization of local Gorenstein-rings by its injective dimension .....	320
Chapter VIII - GENERALIZED COHEN-MACAULAY RINGS AND BLOWING UP .....	326
§37 Finiteness of local cohomology .....	326
§38 Standard system of parameters .....	335
§39 The computation of local cohomology of generalized Cohen-Macaulay rings .....	350
§40 Blowing up of a standard system of parameters .....	353
§41 Standard ideals on Buchsbaum rings .....	367
§42 Examples .....	390

Chapter IX – APPLICATIONS OF LOCAL COHOMOLOGY TO THE  
COHEN-MACAULAY BEHAVIOUR OF BLOWING UP RINGS ..... 397

§43	Generalized Cohen-Macaulay rings with respect to an ideal .....	397
§44	The Cohen-Macaulay property of Rees algebras .....	400
§45	Rees algebras of $\mathfrak{m}$ -primary ideals .....	404
§46	The Rees algebra of parameter ideals .....	415
§47	The Rees algebra of powers of parameter ideals .....	418
§48	Applications to rings of low multiplicity .....	421
	Examples .....	422

Appendix (B. Moonen) – GEOMETRIC EQUIMULTIPLICITY

INTRODUCTION .....	448
I. LOCAL COMPLEX ANALYTIC GEOMETRY .....	452
§ 1. Local analytic algebras .....	453
1.1. Formal power series .....	453
1.2. Convergent power series .....	454
1.3. Local analytic $\mathbb{k}$ -algebras .....	456
§ 2. Local Weierstraß Theory I: The Division Theorem .....	458
2.1. Ordering the monomials .....	458
2.2. Monomial ideals and leitideals .....	459
2.3. The Division Theorem .....	461
2.4. Division with respect to an ideal; standard bases .....	466
2.5. Applications of standard bases: the General Weierstraß Preparation Theorem and the Krull Intersection Theorem .....	467
2.6. The classical Weierstraß Theorems .....	468
§ 3. Complex spaces and the Equivalence Theorem .....	469
3.1. Complex spaces .....	470
3.2. Constructions in <u>cpl</u> .....	474
3.3. The Equivalence Theorem .....	477
3.4. The analytic spectrum .....	480
§ 4. Local Weierstraß Theory II: Finite morphisms .....	481
4.1. Finite morphisms .....	482
4.2. Weierstraß maps .....	482
4.3. The Finite Mapping Theorem .....	484
4.4. The Integrality Theorem .....	488