

Contents

General notation	xi
0 Introduction	1
0.1 How to read this book	1
0.2 What is K-theory?	2
0.3 Brutal synopsis of the book	8
PART I C*-ALGEBRAS	
1 C*-algebras—a summary	15
1.1 C*-algebras	15
1.2 Units and approximate units	16
1.3 Spectral theory	17
1.4 Ideals	18
1.5 Operator algebras and representations	18
1.6 Positivity	20
1.7 Infinite sums	20
1.8 Inductive limits	21
1.9 Tensor products	21
1.10 Stabilization	22
1.11 Non-commutative topology	24
1.12 Exercises	25
2 Multiplier algebras	26
2.1 Unitizations	27
2.2 Multipliers and double centralizers	31
2.3 Strict topology	39
2.4 Exercises	44
3 Extensions of C*-algebras	51
3.1 Exact sequences	52
3.2 Extensions and the Busby invariant	55
3.3 Sums of extensions	61
3.4 Exercises	65

4	Invertibles and unitaries	73
4.1	Normalized matrices	73
4.2	Homotopies	75
4.3	Lifting results	80
4.4	Inductive limits	82
4.5	Exercises	84
5	Projections	85
5.1	Partial isometries and projections	85
5.2	Equivalence of projections	88
5.3	Decomposition into matrix algebras	93
5.4	Exercises	100
 PART II FUNDAMENTALS OF K-THEORY		
6	K_0—basic properties	107
6.1	The monoid $V(A)$	109
6.2	The group $K_0(A)$	110
6.3	Exact sequences	119
6.4	Homotopy invariance	121
6.5	Catalogue of basic K-groups	123
6.6	Exercises	124
7	K_1 and suspensions	130
7.1	The K-group $K_1(A)$	130
7.2	Suspended C*-algebras	136
7.3	Exercises	144
8	The index map in K-theory	146
8.1	Definition of the connecting map	146
8.2	The long exact sequence	149
8.3	Exercises	154
9	Bott periodicity	158
9.1	The Bott map	159
9.2	Approximation by loops	161
9.3	The cyclic six term exact sequence	169
9.4	Exercises	172
10	K-theory for multiplier algebras	180

11 Homology	184
11.1 Homology functors	184
11.2 Bott periodicity and functorial K-theory	190
11.3 Exercises	195
12 Some examples: AF-algebras, Cuntz algebras, rotation algebras	196
12.1 AF-algebras	196
12.2 The Cuntz algebras \mathcal{O}_n	201
12.3 The (irrational) rotation algebras A_θ	202
12.4 Exercises	206
13 Vector bundles and topological K-theory	210
13.1 Vector bundles and the semigroup $V(X)$	211
13.2 Functoriality	213
13.3 The K-groups	214
13.4 Sections	217
13.5 Exercises	219

PART III HILBERT MODULES AND A GENERALIZED INDEX THEORY

14 The classical Fredholm index	225
14.1 The index and its properties	225
14.2 Exercises	229
15 Hilbert modules	231
15.1 Hilbert modules	232
15.2 Adjointable operators	240
15.3 Complementability and polar decompositions	248
15.4 Finitely and countably generated projective modules	254
15.5 Exercises	259
16 The Kuiper–Mingo theorem	269
17 A generalized Fredholm index	277
17.1 The generalized Atkinson theorem	278
17.2 Definition of the generalized Fredholm index	281
17.3 Properties of the index map	285

PART IV APPENDICES

G The Grothendieck group	295
L Inductive limits	298
L.1 Algebraic direct limits	298
L.2 C*-algebraic inductive limits	302
O (Dis)order and positivity in C*-algebras	305
T Tensor products—or: the importance of being subcross	309
T.1 Introduction	309
T.2 Algebraic tensor products	313
T.3 Norms on algebraic tensor products	323
T.4 Tensor product of Hilbert spaces	328
T.5 The spatial tensor product	331
T.6 Other C*-algebra tensor products	343
T.7 Exercises	356
References	362
Subject index	367