## **CONTENTS**

I The Center of Mass

1.1 Introduction, 1

	1.3	Physical Motivations in Geometry, 3	
	1.4	Further Physical Motivations, 7	
	1.5	An Axiomatic Characterization of Center of Mass, 12	
	1.6	An Algebraic Attack on Geometry, 18	
	1.7	Painting a Triangle, 24	
	1.8	Barycentric Coordinates, 27	
	1.9		
	1.10	Affine Geometry, 34	
2	Vec	tor Algebra	3
2		Introduction, 36	3
2	2.1		3
2	2.1 2.2	Introduction, 36	3
2	2.1 2.2 2.3	Introduction, 36 The Definition of Vector, 37	3
2	2.1 2.2 2.3	Introduction, 36 The Definition of Vector, 37 Vector Addition, 40 Scalar Multiplication, 44	3
2	2.1 2.2 2.3 2.4 2.5	Introduction, 36 The Definition of Vector, 37 Vector Addition, 40 Scalar Multiplication, 44	3

1.2 Some Physical Assumptions and Conventions, 1

2.7 A Vector Approach to the Center of Mass, 57

3	Vec	tor Spaces and Subspaces	63
	3.2 3.3 3.4 3.5 3.6	Further Examples, 77 Affine Subspaces, 79 Some Separation Theorems, 85 Some Collinearity and Concurrence Theorems, 91	
4	Len	gth and Angle	97
	4.3 4.4 4.5	Introduction, 97 Geometric Definition of the Inner Product, 97 Proofs Involving the Inner Product, 102 The Metric Axioms, 107 Some Analytic Geometry, 112 Orthogonal Subspaces, 119 Skew Coordinates, 123	
5	Mis	cellaneous Applications	131
	5.2 5.3 5.4 5.5 5.6	Introduction, 131 The Method of Orthogonal Projections, 131 Linear Equations: Three Views, 138 A Useful Formula, 145 Motion, 152 A Minimum Principle, 161 Function Spaces, 164	
6	Ar	ea and Volume	169
	6.2 6.3 6.4 6.5	Introduction, 169 Area in the Plane: An Axiom System, 171 Area in the Plane: A Vector Formulation, 176 Area of Polygons, 180 Further Examples, 186 Volumes in 3-Space, 190	

	6.7	Area Equals Base Times Height, 197	
	6.8		
	6.9	Vector Areas, 209	
7	E	ther Generalizations	214
•	rur	Lifer Generalizations	217
	7.1	Introduction, 214	
	7.2		
	7.3	Some Theorems on Determinants, 221	
		Even and Odd Permutations, 226	
	7.5	Outer Products in n-Space, 228	
	7.6	Some Topology, 233	
	7.7	Areas of Curved Figures, 244	
8	Mat	rices and Linear Transformations	254
	8.1	Introduction, 254	
	8.2	Some Examples, 255	
	8.3	Affine and Linear Transformations, 260	
	8.4	The Matrix of a Linear Transformation, 265	
	8.5	The Matrix of an Affine Transformation, 277	
	8.6	Translations and Dilatations, 281	
	8.7	The Reduction of an Affine Transformation to a Linear One, 285	
	8.8	A Fixed Point Theorem with Probabilistic Implications, 291	
_			207
9	Are	a and Metric Considerations	297
	9.1	Introduction, 297	
	9.2	Determinants, 297	
	9.3	Applications to Analytic Geometry, 304	
	9.4	Orthogonal and Euclidean Transformations, 307	
	9.5	Classification of Motions of the Plane, 314	
	9.6	Classification of Motions of 3-Space, 318	
10	The	Algebra of Matrices	323
		Introduction, 323	
	10.2	Multiplication of Matrices, 323	
		Inverses, 331	
	10.4	The Algebra of Matrices, 339	
	10.5		
	10.6		
	10.7	Projections and Reflections 355	

362

393

II Groups

Index

11.1	Introduction, 362
11.2	Definitions and Examples, 362
11.3	The "Erlangen Program," 369
	Symmetry, 374
	Physical Applications of Symmetry, 379
	Abstract Groups, 384