

Contents

Introduction	1
--------------	---

I. FUNDAMENTALS ON SESQUILINEAR FORMS

1. Orthosymmetric sesquilinear forms	4
1.1 The underlying division rings p. 4 - 1.2 The concept of sesquilinear form p. 5 - 1.3 Orthosymmetric forms are ε -her- mitean p. 6 - 1.4 Zoology of forms p. 10 - 1.5 Scaling of forms p. 10 - 1.6 Existence of ε -hermitean forms p. 11	
2. Trace-valued forms and hyperbolic planes	13
3. Positive forms	16
4. Dense subspaces	19
5. Finite dimensional subspaces	20
6. Closed subspaces	23
7. Isometries between sesquilinear spaces	28
8. The weak linear topology $\sigma(\phi)$ on (E, ϕ)	33
9. Orthostable lattices of subspaces	39
References	46
Appendix I. A division ring which admits no sesquilinear form and a remark on Baer orderability	50
1. Dickson's Example p. 50 - 2. There is no antiautomorphism p. 53 - 3. Modifying Dickson's Example p. 55 - 4. Baer ordered *-fields p. 57	

II. DIAGONALIZATION OF K_0 -FORMS

1. Introduction	61
2. Diagonalization	63
3. Stability (Definition)	67
4. A stable form is determined by the elements it represents	70
5. Quasistability	74
6. Weak stability	76
7. A lemma on supplements	76
References	81

Appendix I. A few examples of "suitable" fields	83
0. Introduction	83
1. Commutative nonformally real fields (characteristic $\neq 2$)	83
1.1 Nonformally real fields k with finite multiplicative group k/k^2 p. 84 - 1.2 The C_i -fields p. 85	
2. Commutative formally real fields	86
3. Commutative fields in characteristic 2	87
4. Involutorial division rings suitable for isotropic hermitean forms	88
5. A formally real involutorial division ring	89

III. WITT DECOMPOSITIONS FOR HERMITEAN N_0 -FORMS

1. Introduction	96
2. The lattice that belongs to the problem	97
3. Metabolic decompositions	99
4. A lemma on orthogonal separation of totally isotropic subspaces	101
5. Reducing the proof of Thm. 2 to the case of a nondegenerate E^*	102
6. Discussion of properties (1) and (2) when E^* is nondegenerate	103
7. Proof of Theorem 2 when E^* is nondegenerate	105
8. Some general remarks on the proof of Theorem 2	107
References	109

IV. ISOMORPHISMS BETWEEN LATTICES OF LINEAR SUBSPACES WHICH ARE INDUCED BY ISOMETRIES

1. Introduction	110
2. The kind of lattices admitted	110
3. Statement of Theorem 1 and an outlay of its proof	113
4. The construction problem	114
5. Solution of the construction problem in the irreducible case	116
6. Solution of the construction problem in the reducible case (end of the proof of Theorem 1)	119
7. Remarks on the case of not complete sublattices	119
8. Non-alternate forms: Theorem 2	120
9. Proof of Theorem 2	122
10. Remarks on the method	124
References	126

V. SUBSPACES IN TRACE - VALUED SPACES WITH MANY ISOTROPIC VECTORS

1. Introduction	127
2. Classification of a single subspace	128
3. An application to Witt decompositions	131
4. Remarks on canonical bases	133
References	135

VI. ORTHOGONAL AND SYMPLECTIC SEPARATION

1. Introduction	136
2. On the lattice $V(F,G)$ of an orthogonal pair	137
3. Orthogonal separation in trace - valued spaces	141
4. Symplectic separation in trace - valued spaces	145
References	149

VII. CLASSIFICATION OF HERMITEAN FORMS IN CHARACTERISTIC TWO

1. Introduction	151
2. Multiples of rigid spaces	152
3. The relation \sim on the forms of countable dimension	155
4. Weakly stable spaces	157
5. Fitting together stable and rigid spaces	160
6. The classification of weakly stable spaces	162
7. Representatives	166
8. Suitable fields for weak stability	166
References	168

VIII. SUBSPACES IN NON - TRACE - VALUED SPACES

1. Introduction	169
2. The lattice of a totally isotropic subspace ($\dim S/T < \infty$)	170
3. Remarks on the verification of diagrams	172
4. Totally isotropic subspaces: the indices	174
5. Totally isotropic subspaces: the irreducible objects	177
6. The invariants of a totally isotropic subspace	181
7. The decomposition theorem	185
8. On closed totally isotropic subspaces	191
9. The case of Witt decompositions reviewed	195
10. Remarks on related results (Principle II)	197
References	201

IX. INVOLUTIONS IN HERMITEAN SPACES IN CHARACTERISTIC TWO

1. Introduction	202
2. The form derived from an involution	203
3. Orthogonal similarity	204
4. A special case	204
5. A lattice material to the solution of the general problem	207
6. Remarks on the lattice	211
7. The classification problem	214
8. Remarks on the proof of the classification problem	216
9. On the classification of nilpotent self-adjoint transformations	220
10. Canonical representatives	222
References	224

X. EXTENSION OF ISOMETRIES

0. Introduction	225
1. Recall of dual pairs (algebraic formulation)	226
2. Topological setting	229
3. Mackey's theorem on modular pairs	231
4. Isometries between dense subspaces	234
5. Isometries between closed subspaces	236
6. Isometries between arbitrary subspaces	239
7. The results of Chapter VI as an inference from Theorem 5 in Section 5	243
8. Transgression into the uncountable: an application of the log frame	245
9. On the extension of algebraic isometries	248
References	252

XI. CLASSIFICATION OF FORMS OVER ORDERED FIELDS

1. Introduction	253
2. Weakly isotropic forms	255
3. Examples of fields in connection with properties (1) and (2)	257
4. A remark on Hilbert ordered skew-fields	258
5. Two Hasse Principles	259
6. The classification	261
7. Canonical representatives for quasistable forms	262
8. Fields over which all K_0 -forms are quasistable	265
References	267

XII. CLASSIFICATION OF SUBSPACES IN SPACES WITH DEFINITE FORMS

0. Introduction	269
1. Standard bases for \perp -dense subspaces and their matrices	270
2. The matrix of a \perp -dense subspace with standard basis	273
3. The ψ -invariant of a \perp -dense subspace	276
4. The Main Theorem on \perp -dense subspaces and the plan of its proof	278
5. Proof of the Main Theorem: the first lemma	279
6. Proof of the Main Theorem: the second lemma	282
7. End of the proof of the Main Theorem: the third lemma	287
8. An important special case: \perp -dense hyperplanes	293
Appendix I. An interpretation of the invariant ψ in Sec. 3	295
Appendix II. The proof of a theorem in Section 2	299
9. Standard bases for arbitrary subspaces (Definitions and existence)	304
10. The matrices associated with a standard basis	306
11. The Main Theorem on arbitrary subspaces (Statement)	309
12. The proof	312
13. Embeddings that split	314
14. Conditions for \perp -dense embeddings to split	316
15. Conditions for \perp -closed embeddings to split	318
16. Parseval embeddings	320
References	327

XIII. CLASSIFICATION OF \perp -DENSE SUBSPACES WITH DEFINITE FORMS

1. Introduction	328
2. Digression on Lagrange's identity	330
3. The example " $\dim E/V = 2$ ". The invariant $\{\Omega_{ij}\}$	333
4. A change of the basis in V does not affect the Ω_{ij}	335
5. Transformation formula for D_n	338
6. Transformation law for the quantities Ω_{ij}	340
7. The main theorem ($\dim E/V = 2$; $D_n \rightarrow \infty$)	340
8. Embeddings ($\dim E/V = 2$; $D_n \rightarrow \infty$) that split	343
9. An application to dense embeddings when $k_0 = \mathbb{R}$	345
10. Counting orbits of \perp -dense subspaces in arbitrary hermitian spaces over \mathbb{R} , \mathbb{C} or \mathbb{H}	351
11. Applications to the theory of divergent series	352
References	353

XIV. QUADRATIC FORMS

0. Introduction	354
1. Symmetrization	356
2. The process of squaring	357
3. The concept of quadratic form	358
4. Isometries between quadratic spaces	361
5. A remark on forms in characteristic two	366
References	368
Appendix I. The dimension of S/T and a theorem on division algebras in characteristic 2	370

XV. WITT'S THEOREM IN FINITE DIMENSIONS

1. Introduction	375
2. Witt's Theorem for finite dimensional quadratic forms and for trace-valued sesquilinear forms	376
3. A Witt type theorem for finite dimensional non-trace-valued sesquilinear forms	380
References	385

XVI. ARF'S THEOREM IN DIMENSION \aleph_0

1. Introduction	387
2. Glauser's lattice	388
3. Invariants of a subspace	391
4. Characterization of the orbit of a subspace (Theorem 2)	393
5. Proof of Theorem 2: Construction of the initial triple $(\tilde{\tau}_0, W_0, \bar{W}_0)$	396
6. Proof of Theorem 2: The general step in Case I	398
7. Proof of Theorem 2: Case II	401
8. The irreducible objects	404
References	406
Appendix I. Quaternions in characteristic 2 and a remark on the Arf invariant à la Tits	407
Symbols and Notations	414
Index of Names	415
Index	417