

Contents

Chapter 1	
Introduction	1
1.1. Mean Curvature	1
1.2. Laplace's Equation	3
1.3. Angle of Contact	4
1.4. The Method of Gauss; Characterization of the Energies	4
1.5. Variational Considerations	7
1.6. The Equation and the Boundary Condition	10
1.7. Divergence Structure	11
1.8. The Problem as a Geometrical One	11
1.9. The Capillary Tube	12
1.10. Dimensional Considerations	13
Notes to Chapter 1	14
Chapter 2	
The Symmetric Capillary Tube	17
2.1. Historical and General	17
2.2. The Narrow Tube; Center Height	18
2.3. The Narrow Tube; Outer Height	22
2.4. The Narrow Tube; Estimates Throughout the Trajectory	24
2.5. Height Estimates for Tubes of General Size	25
2.6. Meniscus Height; Narrow Tubes	29
2.7. Meniscus Height; General Case	30
2.8. Comparisons with Earlier Theories	32
Notes to Chapter 2	35
Chapter 3	
The Symmetric Sessile Drop	37
3.1. The Correspondence Principle	37
3.2. Continuation Properties	38
3.3. Uniqueness and Existence	40
3.4. The Envelope	42
3.5. Comparison Theorems	43
3.6. Geometry of the Sessile Drop; Small Drops	50
3.7. Geometry of the Sessile Drop; Larger Drops	60
Notes to Chapter 3	65

Chapter 4

The Pendent Liquid Drop	67
4.1. Mise en Scène	67
4.2. Local Existence	67
4.3. Uniqueness	69
4.4. Global Behavior; General Remarks	70
4.5. Small $ u_0 $	71
4.6. Appearance of Vertical Points	77
4.7. Behavior for Large $ u_0 $	82
4.8. Global Behavior	86
4.9. Maximum Vertical Diameter	90
4.10. Maximum Diameter	93
4.11. Maximum Volume	95
4.12. Asymptotic Properties	96
4.13. The Singular Solution	100
4.14. Isolated Character of Global Solutions	102
4.15. Stability	104
Notes to Chapter 4	106

Chapter 5

Asymmetric Case; Comparison Principles and Applications	110
5.1. The General Comparison Principle	110
5.2. Applications	113
5.3. Domain Dependence	122
5.4. A Counterexample	124
5.5. Convexity	128
Notes to Chapter 5	130

Chapter 6

Capillary Surfaces Without Gravity	133
6.1. General Remarks	133
6.2. A Necessary Condition	134
6.3. Sufficiency Conditions	140
6.4. Sufficiency Conditions II	144
6.5. A Subsidiary Extremal Problem	147
6.6. Minimizing Sequences	147
6.7. The Limit Configuration	148
6.8. The First Variation	150
6.9. The Second Variation	154
6.10. Solution of the Jacobi Equation	155
6.11. Convex Domains	160
6.12. Continuous and Discontinuous Disappearance	163
6.13. An Example	164
6.14. Another Example	165
6.15. Remarks on the Extremals	166
6.16. Example 1	168
6.17. Example 2	169
6.18. Example 3	170
6.19. The Trapezoid	171
6.20. Tail Domains; A Counterexample	183

6.21. Convexity	184
6.22. A Counterexample	185
6.23. Transition to Zero Gravity	185
Notes to Chapter 6	187
 Chapter 7	
Existence Theorems	189
7.1. Choice of Venue	189
7.2. Variational Solutions	191
7.3. Generalized Solutions	192
7.4. Construction of a Generalized Solution	193
7.5. Proof of Boundedness	196
7.6. Uniqueness	201
7.7. The Variational Condition; Limiting Case	203
7.8. A Necessary and Sufficient Condition	205
7.9. A Limiting Configuration	206
7.10. The Case $\mu > \mu_0 > 1$	207
7.11. Application: A General Gradient Bound	208
Notes to Chapter 7	210
 Chapter 8	
The Capillary Contact Angle	212
8.1. Everyday Experience	212
8.2. The Hypothesis	213
8.3. The Horizontal Plane; Preliminary Remarks	214
8.4. Necessity for φ	214
8.5. Proof that γ is Monotone	214
8.6. Geometrically Imposed Stability Bounds	218
8.7. A Further Kind of Instability	219
8.8. The Inclined Plane; Preliminary Remarks	220
8.9. Integral Relations, and Impossibility of Constant Contact Angle	221
8.10. The Zero-Gravity Solution	222
8.11. Postulated Form for φ	223
8.12. Formal Analytical Solution	224
8.13. The Expansion; Leading Terms	225
8.14. Computer Calculations	227
8.15. Discussion	228
8.16. Further Discussion	229
Notes to Chapter 8	232
 Chapter 9	
Identities and Isoperimetric Relations	234
 Bibliography	