

CONTENTS

1.	DIMENSIONAL ANALYSIS AND SCALING	1
1.1	Dimensional Analysis / 1 <i>The Program of Applied Mathematics / 1</i> <i>Dimensional Methods / 3</i> <i>Exercises / 4</i>	
1.2	The Buckingham Pi Theorem / 5 <i>Formulation / 5</i> <i>Application to a Diffusion Problem / 7</i> <i>Proof of the Pi Theorem / 10</i> <i>Exercises / 14</i>	
1.3	Scaling / 16 <i>Characteristic Scales / 16</i> <i>Heat Conduction / 18</i> <i>The Projectile Problem / 21</i> <i>Scaling Known Functions / 26</i> <i>Exercises / 30</i> <i>References / 33</i>	
2.	PERTURBATION METHODS	34
2.1	Regular Perturbation / 34 <i>The Perturbation Method / 34</i> <i>Motion in a Nonlinear Resistive Medium / 36</i>	

<i>A Nonlinear Oscillator</i>	/ 39
<i>The Poincaré-Lindstedt Method</i>	/ 42
<i>Asymptotics</i>	/ 44
<i>Exercises</i>	/ 49
2.2 Singular Perturbation	/ 53
<i>Failure of Regular Perturbation</i>	/ 53
<i>Inner and Outer Approximations</i>	/ 54
<i>Algebraic Equations and Balancing</i>	/ 58
<i>Exercises</i>	/ 59
2.3 Boundary Layer Analysis	/ 60
<i>The Inner Approximation</i>	/ 60
<i>Matching</i>	/ 62
<i>Uniform Approximations</i>	/ 63
<i>A Worked Example</i>	/ 64
<i>Boundary Layer Phenomena</i>	/ 66
<i>Exercises</i>	/ 69
2.4 Two Applications	/ 70
<i>Damped Harmonic Oscillator</i>	/ 71
<i>A Chemical Kinetics Problem</i>	/ 76
<i>Exercises</i>	/ 83
<i>References</i>	/ 84

3. CALCULUS OF VARIATIONS

85

3.1 Variational Problems	/ 85
<i>Functionals</i>	/ 85
<i>Examples</i>	/ 87
<i>Exercises</i>	/ 90
3.2 Necessary Conditions for Extrema	/ 90
<i>Normed Linear Spaces</i>	/ 90
<i>Derivatives of Functionals</i>	/ 95
<i>Necessary Conditions</i>	/ 98
<i>Exercises</i>	/ 99
3.3 The Simplest Problem	/ 101
<i>The Euler Equation</i>	/ 101
<i>Solved Examples</i>	/ 104
<i>First Integrals</i>	/ 106
<i>Exercises</i>	/ 108
3.4 Generalizations	/ 110
<i>Higher Derivatives</i>	/ 110

<i>Several Functions / 112</i>	
<i>Multiple Integral Problems / 114</i>	
<i>Natural Boundary Conditions / 118</i>	
<i>Exercises / 122</i>	
3.5 Hamiltonian Theory / 126	
<i>Hamilton's Principle / 126</i>	
<i>Hamilton's Principle Versus Newton's Law / 132</i>	
<i>The Canonical Formalism / 133</i>	
<i>The Inverse Problem / 137</i>	
<i>Exercises / 140</i>	
3.6 Isoperimetric Problems / 143	
<i>Necessary Conditions / 143</i>	
<i>Exercises / 148</i>	
<i>References / 150</i>	
4. EQUATIONS OF APPLIED MATHEMATICS	151
4.1 Partial Differential Equations / 151	
<i>Definitions / 151</i>	
<i>Linearity versus Nonlinearity / 155</i>	
<i>Superposition / 157</i>	
<i>Exercises / 158</i>	
4.2 The Diffusion Equation / 159	
<i>Heat Conduction / 159</i>	
<i>Well-Posed Problems / 162</i>	
<i>Diffusion in Higher Dimensions / 166</i>	
<i>Exercises / 172</i>	
4.3 Classical Techniques / 174	
<i>Separation of Variables / 174</i>	
<i>Fourier Series / 178</i>	
<i>Sturm-Liouville Problems / 182</i>	
<i>Integral Transforms / 188</i>	
<i>Exercises / 195</i>	
4.4 Integral Equations / 199	
<i>Classification and Origins / 199</i>	
<i>Relationship to Differential Equations / 204</i>	
<i>Fredholm Equations / 208</i>	
<i>Symmetric Kernels / 212</i>	
<i>Volterra Equations / 216</i>	
<i>Exercises / 224</i>	
<i>References / 229</i>	

5. WAVE PHENOMENA IN CONTINUOUS SYSTEMS	230
5.1 Wave Propagation / 230	
<i>Waves</i> / 230	
<i>Linear Waves</i> / 234	
<i>Nonlinear Waves</i> / 237	
<i>Burgers' Equation</i> / 242	
<i>The Korteweg-deVries Equation</i> / 245	
<i>Conservation Laws</i> / 248	
<i>Quasi-Linear Equations</i> / 253	
<i>Exercises</i> / 260	
5.2 Mathematical Models of Continua / 264	
<i>Kinematics</i> / 264	
<i>Mass Conservation</i> / 270	
<i>Momentum Conservation</i> / 272	
<i>Thermodynamics and Energy Conservation</i> / 276	
<i>The Acoustic Approximation</i> / 282	
<i>Stress Waves in Solids</i> / 284	
<i>Exercises</i> / 288	
5.3 The Wave Equation / 292	
<i>D'Alembert's Solution</i> / 292	
<i>Scattering and Inverse Problems</i> / 298	
<i>Exercises</i> / 301	
5.4 Gasdynamics / 305	
<i>Conservation Laws</i> / 305	
<i>Riemann's Method</i> / 307	
<i>The Rankine-Hugoniot Conditions</i> / 313	
<i>Exercises</i> / 315	
5.5 Fluid Motions in R^3 / 316	
<i>Kinematics</i> / 316	
<i>Dynamics</i> / 323	
<i>Energy</i> / 331	
<i>Exercises</i> / 335	
<i>References</i> / 340	
6. STABILITY AND BIFURCATION	341
6.1 Intuitive Ideas / 342	
<i>Stability and Population Dynamics</i> / 342	
<i>Bifurcation of a Bead on a Hoop</i> / 346	

<i>Stability and Chemotaxis of Amoebae /</i>	351
<i>Exercises /</i>	357
6.2 One Dimensional Problems /	357
<i>Stability /</i>	357
<i>Classification of Bifurcation Points /</i>	364
<i>Exchange of Stability /</i>	368
<i>Continuously Stirred Tank Reactor /</i>	373
<i>Exercises /</i>	377
6.3 Two Dimensional Problems /	378
<i>Phase Plane Phenomena /</i>	378
<i>Linear Systems /</i>	385
<i>Nonlinear Systems /</i>	391
<i>Bifurcation /</i>	394
<i>Exercises /</i>	404
6.4 Hydrodynamic Stability /	406
<i>A Layered Fluid /</i>	406
<i>Perturbation Equations /</i>	408
<i>Rayleigh's Example /</i>	410
<i>Exercises /</i>	414
<i>References /</i>	415

7. SIMILARITY METHODS	416
7.1 Invariant Variational Problems /	418
<i>Local Lie Groups /</i>	418
<i>Invariance of Functionals /</i>	423
<i>The Noether Theorem /</i>	429
<i>Conservation Laws in Mechanics /</i>	435
<i>Exercises /</i>	437
7.2 Invariant Partial Differential Equations /	441
<i>Self-Similar Solutions /</i>	441
<i>Similarity and Dimensional Analysis /</i>	444
<i>The Method of Stretchings /</i>	447
<i>The Lie Plane /</i>	457
<i>Exercises /</i>	461
7.3 The General Similarity Method /	464
<i>Local Lie Groups on R^3 /</i>	464
<i>Invariant Partial Differential Equations /</i>	469
<i>The Determination of Symmetries /</i>	474
<i>Self-Similar Solutions /</i>	476

<i>Exercises / 486</i>
<i>References / 488</i>

8. DIFFERENCE METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS

489

8.1 Finite Difference Methods / 490
<i>Discretization / 490</i>
<i>Discrete Approximations for Derivatives / 493</i>
<i>Exercises / 496</i>
8.2 The Diffusion Equation / 497
<i>An Explicit Scheme / 497</i>
<i>Truncation Error and Convergence / 500</i>
<i>Implicit Scheme / 503</i>
<i>General Boundary Conditions / 509</i>
<i>Stability / 510</i>
<i>Matrix Stability Analysis / 513</i>
<i>Exercises / 514</i>
8.3 The Laplace Equation / 517
<i>Finite Difference Approximation / 517</i>
<i>Iterative Methods for Linear Systems / 521</i>
<i>The Neumann Problem / 533</i>
<i>Accelerating Iterative Schemes / 540</i>
<i>Exercises / 543</i>
8.4 Hyperbolic Equations / 547
<i>The Wave Equation / 547</i>
<i>Hyperbolic Systems / 552</i>
<i>Characteristic Methods / 558</i>
<i>Conservation Laws / 562</i>
<i>Exercises / 564</i>
<i>References / 566</i>