CONTENTS

Preface		ix
Contributors		
AN INVITATIO	ON TO NONSTANDARD ANALYSIS	1
	Tom Lindstrøm	
INTRODUCTION	N	1
CHAPTER I.	A SET OF HYPERREALS	4
	1 Construction of *R	4
	2 Internal sets and functions	10
	3 Infinitesimal calculus	17
CHAPTER II.	SUPERSTRUCTURES AND LOEB MEASURES	22
	1 Superstructures	22
	2 Loeb measures	27
	3 Brownian motion	38
CHAPTER III	. SATURATION AND TOPOLOGY	48
	1 Beyond R ₁ -saturation	48
	2 General topology	52
	3 Completions, compactifications, and nonstandard hulls	59
CHAPTER IV.	THE TRANSFER PRINCIPLE	68
	1 The languages $L(V(S))$ and $L^*(V(S))$	68
	2 Los' theorem and the transfer principle	73
	3 Axiomatic nonstandard analysis	83
APPENDIX	ULTRAFILTERS	84
NOTES		90
REFERENCES		99

vi CONTENTS

INFINITESIMALS	IN PROBABILITY THEORY	106
	Jerome Keisler	100
1	The hyperfinite time line	107
2	Universal and homogeneous probability spaces	109
3	Stochastic processes	112
4	Products of Loeb spaces	114
5	Liftings of stochastic processes	117
6	Adapted probability spaces	122
7	Adapted distributions	125
8	Universal and homogeneous adapted spaces	131
9	Applications to stochastic analysis	136
	References	139
Infinitesimals	IN FUNCTIONAL ANALYSIS	140
C.	Ward Henson	
1	Topological vector spaces	142
2	Operators	159
3	Uniform equivalence	164
4	Indiscernibles	166
5	Isomorphic nonstandard hulls	175
	References	180
APPLICATIONS OF	F NONSTANDARD ANALYSIS IN MATHEMATICAL PHYSICS	182
	rgio Albeverio	102
1	Introduction	182
2	Singular interactions in Schrödinger operators: a case	
	study	184
3	Nonstandard theory applied to closed bilinear forms on	
	Hilbert spaces	190
4	Dirichlet forms as standard parts of hyperfinite	
	Dirichlet forms, and applications to quantum mechanics	194
5	Hyperfinite energy forms, diffusions and quantum	
	mechanics on fractals	200
6	Polymer measures	203
7	Quantum fields: hyperfinite models and connections	
	with polymer measures	205

CONTENTS		vii
8	Other topics and conclusions	211
	References	213
A LATTICE FOR	MULATION OF REAL AND VECTOR VALUED INTEGRALS	221
P	eter A. Loeb	
1	Scalar functions and measures	222
2	Internal functionals on continuous functions	229
3	Vector functions and measures	232
	References	235
AN APPLICATIO	N OF NONSTANDARD METHODS TO COMPUTATIONAL GROUP THEORY	237
E	. Benninghofen and M.M. Richter	
	Introduction	237
1	Group theoretic preliminaries	237
2	The growth function and automata	240
3	The nonstandard hull of FN(a,b)	243
4	Non-regularity results	248
	References	257
SYNTACTICAL N	ETHODS IN INFINITESIMAL ANALYSIS	258
F	rancine Diener and Keith D. Stroyan	
1	Introduction	258
2	IST: an alternate axiomatization of set theory	259
8	Bounded internal set theory	263
4	The bounded formulas (T), (I), (S).	266
	Quantifier reduction rules	271
6	Generalized transfer and idealization	275
7	Permanence principles	278
	References	280
SOME ASYMPTO	TIC RESULTS IN ORDINARY DIFFERENTIAL EQUATIONS	282
1	rancine and Marc Diener	
1	Properties of the trajectories	283
•	ε-shadows expansions of implicit functions	287
	B Existence of an expansion	291
4	Conclusion: application to the problem of streams	293
	References	295

viii	ONTENTS
------	---------

SUPERINFIN	298		
	Ke	eith D. Stroyan	
	1	Monads	298
	2	Product monads	300
	3	Limit spaces and topologies	303
	4	Limit vector spaces	305
	5	Limit infinitesimals	308
	6	Examples	315
	7	Counterexamples	317
		References	320
THE NON-LI	NEAF	R BOLTZMANN EQUATION FAR FROM EQUILIBRIUM	321
	Le	eif Arkeryd	
	1	Introduction	321
	2	The Boltzmann equation	322
	3	Loeb solutions to the Boltzmann equation	325
	4	The Maxwellian limit	335
		References	340
INDEX			341