

Contents

Part I. Submanifold Theory.

1	Preliminaries	3
1.1	Connections on a vector bundle	3
1.2	Levi-Civita connections	9
1.3	Covariant derivative of tensor fields	15
1.4	Vector fields and differential equations	18
1.5	Lie derivative of tensor fields	22
2	Local geometry of submanifolds	25
2.1	Local invariants of submanifolds	25
2.2	Totally umbilic submanifolds	33
2.3	Fundamental theorem for submanifolds of space forms	38
3	Weingarten surfaces in three dimensional space forms	43
3.1	Constant mean curvature surfaces in $N^3(c)$	44
3.2	Surfaces of R^3 with constant Gaussian curvature	51
3.3	Immersed flat tori in S^3	58
3.4	Bonnet transformations	62
4	Focal points	65
4.1	Height and Euclidean distance functions	65
4.2	The focal points of submanifolds of R^n	69
5	Transformation Groups	72
5.1	G-manifolds	72
5.2	Proper actions	77
5.3	Coxeter groups	79
5.4	Riemannian G-manifolds	84
5.5	Riemannian submersions	89
5.6	Sections	94
5.7	Submanifold geometry of orbits	101
5.8	Infinite dimensional examples	103
6	Isoparametric Submanifolds	106
6.1	Isoparametric maps	107
6.2	Curvature distributions	111
6.3	Coxeter groups associated to isoparametric submanifolds	116
6.4	Existence of Isoparametric polynomial maps	125
6.5	Parallel foliations and The Slice Theorem	130
6.6	Applications to minimal submanifolds	135
7	Proper Fredholm Submanifolds in Hilbert spaces	140
7.1	Proper Fredholm immersions	140
7.2	Isoparametric submanifolds in Hilbert spaces	145

8	Topology of Isoparametric Submanifolds	150
8.1	Tight and taut immersions in R^n	150
8.2	Taut immersions in Hilbert space	152
8.3	Homology of isoparametric submanifolds	156
8.4	Rank 2 isoparametric submanifolds in R^m	160
8.5	Parallel foliations	164
8.6	Convexity theorem	166
8.7	Marked Dynkin diagrams for isoparametric submanifolds	170
Part II. Critical Point Theory.			
9	Elementary critical point theory	181
9.1	Preliminaries	181
9.2	The First Deformation Theorem	186
9.3	The Second Deformation Theorem	192
9.4	Morse Functions	195
9.5	Passing a Critical Level	201
9.6	Morse Theory of Submanifolds	208
9.7	The Morse Inequalities	213
10	Advanced critical point theory	223
10.1	Refined Minimaxing	223
10.2	Linking Type	225
10.2	Bott-Samelson Type	229
11	The Calculus of Variations	233
11.1	Sobolev manifolds of fiber bundle sections	236
11.2	Geodesics	243
11.3	Non-linear eigenvalue problems	252
Appendix 260			
References 263			
Index 269			