

# Contents

## *Chapter 1*   **Constant Forms**

<b>1.1</b>	One-Forms	1
<b>1.2</b>	Two-Forms	5
<b>1.3</b>	The Evaluation of Two-Forms, Pullbacks	8
<b>1.4</b>	Three-Forms	15
<b>1.5</b>	Summary	19

## *Chapter 2*   **Integrals**

<b>2.1</b>	Non-Constant Forms	22
<b>2.2</b>	Integration	24
<b>2.3</b>	Definition of Certain Simple Integrals. Convergence and the Cauchy Criterion	29
<b>2.4</b>	Integrals and Pullbacks	38
<b>2.5</b>	Independence of Parameter	44
<b>2.6</b>	Summary. Basic Properties of Integrals	49

## *Chapter 3*   **Integration and Differentiation**

<b>3.1</b>	The Fundamental Theorem of Calculus	52
<b>3.2</b>	The Fundamental Theorem in Two Dimensions	58
<b>3.3</b>	The Fundamental Theorem in Three Dimensions	65
<b>3.4</b>	Summary. Stokes Theorem	72

## *Chapter 4*   **Linear Algebra**

<b>4.1</b>	Introduction	76
<b>4.2</b>	Constant $k$ -Forms on $n$ -Space	86

<b>4.3</b>	Matrix Notation. Jacobians	94
<b>4.4</b>	The Implicit Function Theorem for Affine Maps	105
<b>4.5</b>	Abstract Vector Spaces	113
<b>4.6</b>	Summary. Affine Manifolds	127

## *Chapter 5*    **Differential Calculus**

<b>5.1</b>	The Implicit Function Theorem for Differentiable Maps	132
<b>5.2</b>	$k$ -Forms on $n$ -Space. Differentiable Maps	142
<b>5.3</b>	Proofs	151
<b>5.4</b>	Application: Lagrange Multipliers	160
<b>5.5</b>	Summary. Differentiable Manifolds	190

## *Chapter 6*    **Integral Calculus**

<b>6.1</b>	Summary	196
<b>6.2</b>	$k$ -Dimensional Volume	197
<b>6.3</b>	Independence of Parameter and the Definition of $\int_S \omega$	200
<b>6.4</b>	Manifolds-with-Boundary and Stokes' Theorem	214
<b>6.5</b>	General Properties of Integrals	219
<b>6.6</b>	Integrals as Functions of $S$	224

## *Chapter 7*    **Practical Methods of Solution**

<b>7.1</b>	Successive Approximation	226
<b>7.2</b>	Solution of Linear Equations	235
<b>7.3</b>	Newton's Method	242
<b>7.4</b>	Solution of Ordinary Differential Equations	245
<b>7.5</b>	Three Global Problems	256

## *Chapter 8*    **Applications**

<b>8.1</b>	Vector Calculus	265
<b>8.2</b>	Elementary Differential Equations	270
<b>8.3</b>	Harmonic Functions and Conformal Coordinates	278
<b>8.4</b>	Functions of a Complex Variable	289
<b>8.5</b>	Integrability Conditions	313

<b>8.6</b>	Introduction to Homology Theory	320
<b>8.7</b>	Flows	328
<b>8.8</b>	Applications to Mathematical Physics	333

**Chapter 9 Further Study of Limits**

<b>9.1</b>	The Real Number System	357
<b>9.2</b>	Real Functions of Real Variables	381
<b>9.3</b>	Uniform Continuity and Differentiability	387
<b>9.4</b>	Compactness	392
<b>9.5</b>	Other Types of Limits	399
<b>9.6</b>	Interchange of Limits	407
<b>9.7</b>	Lebesgue Integration	426
<b>9.8</b>	Banach Spaces	447

	Appendices	456
--	------------	-----

	Answers to Exercises	468
--	----------------------	-----

	Index	504
--	-------	-----