

Contents

Preface	v
Notes to the Reader	xv
1 General Historical Remarks	1
2 The Contents of the <i>Elements</i>	3
2.1 Book I: Foundations of Plane Geometry	3
2.2 Book II: The Geometry of Rectangles	4
2.3 Book III: The Geometry of the Circle	5
2.4 Book IV: Regular Polygons in Circles	5
2.5 Book V: The General Theory of Magnitudes in Proportion	5
2.6 Book VI: The Plane Geometry of Similar Figures . . .	6
2.7 Book VII: Basic Arithmetic	7
2.8 Book VIII: Numbers in Continued Proportion	7
2.9 Book IX: Numbers in Continued Proportion; the Theory of Even and Odd Numbers, Perfect Numbers	8
2.10 Book X: Incommensurable Line Segments	8
2.11 Book XI: Foundations of Solid Geometry	9

2.12	Book XII: Areas and Volumes; Eudoxus's Method of Exhaustion	9
2.13	Book XIII: The Platonic Solids	10
3	The Origin of Mathematics 1: The Testimony of Eudemos	11
	Time Table.	16
4	Euclid Book I	17
4.1	The Overall Composition of Book I	17
4.2	Definitions and Axioms	18
4.3	Book I, Part A: Foundations	21
4.4	Book I, Part B: The Theory of Parallels	31
4.5	Book I, Part C: Parallelograms and Their Areas	36
4.6	Book I, Part D: The Theorem of Pythagoras	42
5	The Origin of Mathematics 2: Parallels and Axioms	47
6	The Origin of Mathematics 3: Pythagoras of Samos	51
7	Euclid Book II	61
8	The Origin of Mathematics 4: Squaring the Circle	73
9	Euclid Book III	79
9.1	The Overall Composition of Book III	79
9.2	The Definitions of Book III	80
9.3	Book III, Part A: Chords in Circles, Circles Intersecting or Touching Each Other	81
9.4	Book III, Part B: Tangents	82
9.5	Book III, Part C ₁ : Angles in Segments of Circles	84
9.6	Book III, Part C ₂ : Chords, Arcs, and Angles	86
9.7	Book III, Part C ₃ : More About Angles in Circles	86
9.8	Book III, Part D: Intersecting Chords, Secants, and Tangents	88

10 The Origin of Mathematics 5: Problems and Theories	93
11 Euclid Book IV	97
11.1 The Contents of Book IV	97
11.2 The Regular Pentagon	99
11.3 Speculations About the Pentagon	102
12 The Origin of Mathematics 6: The Birth of Rigor	109
13 The Origin of Mathematics 7: Polygons After Euclid	113
13.1 What We Missed in Book IV	113
13.2 What Euclid Knew	114
13.3 What Archimedes Did	115
13.4 What Gauss Proved	116
13.5 How Gauss Did It	117
13.6 The Moral of the Story	119
13.7 What Plotinus Has to Say About All This	120
14 Euclid Book V	121
14.1 Proportions Outside of Mathematics	121
14.2 General Remarks About Book V	123
14.3 Proportions in a Modern Version	124
14.4 The Definitions of Book V	127
14.5 The Propositions of Book V	129
15 Euclid Book VI	135
15.1 The Overall Composition of Book VI	135
15.2 The Basis of Similarity Geometry	136
15.3 The Basic Theorems of Similarity Geometry	138
15.4 Book VI, Part C: Proportions and Areas (Products)	141
15.5 Book VI, Part D: Similar Rectilinear Figures	144
15.6 Book VI, Part E: The Application of Areas	146
16 The Origin of Mathematics 8: Be Wise, Generalize	151
16.1 Extending a Result	152
16.2 Weakened Hypotheses	152
16.3 Abstraction	153
16.4 The Case of Pythagoras's Theorem	153

16.5	The Generalization of Ptolemy	155
16.6	Marcel Proust on Abstraction	159
17	Euclid Book VII	161
17.1	The Historical Background	161
17.2	The Overall Composition of Book VII	163
17.3	Definitions	163
17.4	Book VII, Part A: The Euclidean Algorithm	165
17.5	Book VII, Parts B and C: Proportion for Numbers	167
17.6	Book VII, Part D: Proportions and Products	171
17.7	Book VII, Part E: The Greatest Common Divisor and Prime Divisors	176
17.8	Book VII, Part F: The Least Common Multiple	182
18	The Origin of Mathematics 9: Nicomachus and Diophantus	183
18.1	Nicomachus: <i>Introduction to Arithmetic</i>	183
18.2	Diophantus: <i>The Arithmetica</i>	188
19	Euclid Book VIII	193
19.1	The Overall Composition of Books VIII and IX	193
19.2	Book VIII, Part A: Numbers in Continued Proportion	194
19.3	Book VIII, Part B: The Geometry of Numbers	196
20	The Origin of Mathematics 10: Tools and Theorems	203
21	Euclid Book IX	207
21.1	Book IX, Parts B, C, D, and E: More About Numbers in Continued Proportion	207
21.2	Book IX, Part F: The Number of Primes	209
21.3	Book IX, Part G: Odd and Even Numbers, and Perfect Numbers	210

22 The Origin of Mathematics 11: Math Is Beautiful	213
23 Euclid Book X	223
23.1 Commensurability and Its Relation to Other Notions	227
24 The Origin of Mathematics 12: Incommensurability and Irrationality	229
25 Euclid Book XI	255
25.1 The Overall Composition of Book XI	255
25.2 The Definitions of Book XI	256
25.3 Foundations of Solid Geometry	257
25.4 The Affinities of Books I and XI	258
25.5 The Duplication of the Cube	263
26 The Origin of Mathematics 13: The Role of Definitions	267
27 Euclid Book XII	271
27.1 The Overall Composition of Book XII	271
27.2 The Circle	271
27.3 The Pyramid	274
27.4 Cylinders, Cones, and Spheres	278
28 The Origin of Mathematics 14: The Taming of the Infinite	279
29 Euclid Book XIII	283
29.1 The Overall Composition of Book XIII	283
29.2 Division in Extreme and Mean Ratio and the Pentagon	284
29.3 About the Decagon	286
29.4 The Regular Solids	292
29.5 The Classification of the Regular Solids	296

30 The Origin of Mathematics 15: Symmetry Through the Ages	299
30.1 Nature	300
30.2 Art	300
30.3 Philosophy	305
30.4 Mathematics	308
 31 The Origin of Mathematics 16: The Origin of the <i>Elements</i>	 313
 Notes	 317
 Bibliography	 329
 Index	 341