

CONTENTS

PREFACE	vii
CHAPTER 1. EXISTENCE AND UNIQUENESS OF SOLUTIONS.	1
1. Existence of Solutions	1
2. Uniqueness of Solutions	8
3. The Method of Successive Approximations	11
4. Continuation of Solutions	13
5. Systems of Differential Equations	15
6. The n th-order Equation	21
7. Dependence of Solutions on Initial Conditions and Parameters	22
8. Complex Systems	32
Problems	37
CHAPTER 2. EXISTENCE AND UNIQUENESS OF SOLUTIONS (<i>continued</i>)	42
1. Extension of the Idea of a Solution, Maximum and Minimum Solutions	42
2. Further Uniqueness Results	48
3. Uniqueness and Successive Approximations	53
4. Variation of Solutions with Respect to Initial Conditions and Parameters	57
Problems	60
CHAPTER 3. LINEAR DIFFERENTIAL EQUATIONS	62
1. Preliminary Definitions and Notations.	62
2. Linear Homogeneous Systems	67
3. Nonhomogeneous Linear Systems	74
4. Linear Systems with Constant Coefficients	75
5. Linear Systems with Periodic Coefficients	78
6. Linear Differential Equations of Order n	81
7. Linear Equations with Analytic Coefficients	90
8. Asymptotic Behavior of the Solutions of Certain Linear Systems	91
Problems	97
CHAPTER 4. LINEAR SYSTEMS WITH ISOLATED SINGULARITIES: SINGULARITIES OF THE FIRST KIND	108
1. Introduction	108
2. Classification of Singularities	111
3. Formal Solutions	114
4. Structure of Fundamental Matrices	118
5. The Equation of the n th Order	122
6. Singularities at Infinity	127
7. An Example: the Second-order Equation	130
8. The Frobenius Method	132
Problems	135

CHAPTER 5. LINEAR SYSTEMS WITH ISOLATED SINGULARITIES: SINGULARITIES OF THE SECOND KIND	138
1. Introduction	138
2. Formal Solutions	141
3. Asymptotic Series	148
4. Existence of Solutions Which Have the Formal Solutions as Asymptotic Expansions—the Real Case	151
5. The Asymptotic Nature of the Formal Solutions in the Complex Case	161
6. The Case Where A_0 Has Multiple Characteristic Roots	167
7. Irregular Singular Points of an n th-order Equation	169
8. The Laplace Integral and Asymptotic Series	170
Problems	173
CHAPTER 6. ASYMPTOTIC BEHAVIOR OF LINEAR SYSTEMS CONTAINING A LARGE PARAMETER	174
1. Introduction	174
2. Formal Solutions	175
3. Asymptotic Behavior of Solutions	178
4. The Case of Equal Characteristic Roots	182
5. The n th-order Equation	182
Problems	184
CHAPTER 7. SELF-ADJOINT EIGENVALUE PROBLEMS ON A FINITE INTERVAL	186
1. Introduction	186
2. Self-adjoint Eigenvalue Problems	188
3. The Existence of Eigenvalues	193
4. The Expansion and Completeness Theorems	197
Problems	201
CHAPTER 8. OSCILLATION AND COMPARISON THEOREMS FOR SECOND-ORDER LINEAR EQUATIONS AND APPLICATIONS	208
1. Comparison Theorems	208
2. Existence of Eigenvalues	211
3. Periodic Boundary Conditions	213
4. Stability Regions of Second-order Equations with Periodic Coefficients	218
Problems	220
CHAPTER 9. SINGULAR SELF-ADJOINT BOUNDARY-VALUE PROBLEMS FOR SECOND-ORDER EQUATIONS	222
1. Introduction	222
2. The Limit-point and Limit-circle Cases	225
3. The Completeness and Expansion Theorems in the Limit-point Case at Infinity	231
4. The Limit-circle Case at Infinity	242
5. Singular Behavior at Both Ends of an Interval	246
Problems	254
CHAPTER 10. SINGULAR SELF-ADJOINT BOUNDARY-VALUE PROBLEMS FOR n TH-ORDER EQUATIONS	261
1. Introduction	261
2. The Expansion Theorem and Parseval Equality	262

3. The Inverse-transform Theorem and the Uniqueness of the Spectral Matrix	265
4. Green's Function	272
5. Representation of the Spectral Matrix by Green's Function	278
Problems	281
CHAPTER 11. ALGEBRAIC PROPERTIES OF LINEAR BOUNDARY-VALUE PROBLEMS ON A FINITE INTERVAL.	284
1. Introduction	284
2. The Boundary-form Formula	286
3. Homogeneous Boundary-value Problems and Adjoint Problems	288
4. Nonhomogeneous Boundary-value Problems and Green's Function	294
Problems	297
CHAPTER 12. NON-SELF-ADJOINT BOUNDARY-VALUE PROBLEMS	298
1. Introduction	298
2. Green's Function and the Expansion Theorem for the Case $Lx = -x''$	300
3. Green's Function and the Expansion Theorem for the Case $Lx = -x'' + q(t)x$	305
4. The n th-order Case	308
5. The Form of the Expansion	310
Problems	312
CHAPTER 13. ASYMPTOTIC BEHAVIOR OF NONLINEAR SYSTEMS: STABILITY	314
1. Asymptotic Stability	314
2. First Variation: Orbital Stability	321
3. Asymptotic Behavior of a System	327
4. Conditional Stability	329
5. Behavior of Solutions off the Stable Manifold	340
Problems	344
CHAPTER 14. PERTURBATION OF SYSTEMS HAVING A PERIODIC SOLUTION	348
1. Nonautonomous Systems	348
2. Autonomous Systems	352
3. Perturbation of a Linear System with a Periodic Solution in the Nonautonomous Case	356
4. Perturbation of an Autonomous System with a Vanishing Jacobian	364
Problems	370
CHAPTER 15. PERTURBATION THEORY OF TWO-DIMENSIONAL REAL AUTONOMOUS SYSTEMS	371
1. Two-dimensional Linear Systems	371
2. Perturbations of Two-dimensional Linear Systems	375
3. Proper Nodes and Proper Spiral Points	377
4. Centers	381
5. Improper Nodes	384
6. Saddle Points	387
Problems	388
CHAPTER 16. THE POINCARÉ-BENDIXSON THEORY OF TWO-DIMENSIONAL AUTONOMOUS SYSTEMS.	389
1. Limit Sets of an Orbit	389
2. The Poincaré-Bendixson Theorem	391

3. Limit Sets with Critical Points	394
4. The Index of an Isolated Critical Point	398
5. The Index of Simple Critical Point.	400
Problems	402
CHAPTER 17. DIFFERENTIAL EQUATIONS ON A TORUS.	404
1. Introduction	404
2. The Rotation Number	405
3. The Cluster Set	408
4. The Ergodic Case.	409
5. Characterization of Solutions in the Ergodic Case	413
6. A System of Two Equations.	415
REFERENCES.	417
INDEX.	423