

Contents

Preface	ix
1 The Hahn-Hellinger Theorem	
Definitions and the Problem	1
The Case of Multiplicity One, Cyclic Vector	2
Application to Second Order Stochastic Processes	4
Spectral Measures of Higher Multiplicity: A Canonical Example	6
Linear Operators Commuting with Multiplication	7
Spectral Type; Maximal Spectral Type	11
The Hahn-Hellinger Theorem (First Form)	12
The Hahn-Hellinger Theorem (Second Form)	14
Representation of Second Order Stochastic Processes	15
2 The Spectral Theorem for Unitary Operators	
The Spectral Theorem: Multiplicity One Case	17
The Spectral Theorem: Higher Multiplicity Case	18
3 Symmetry and Denseness of the Spectrum	
Spectrum U_T : It is Symmetric	21
Spectrum of U_T : It is Dense	22
Examples	23
4 Multiplicity and Rank	
A Theorem on Multiplicity	27
Approximation with Multiplicity N	28
Rank and Multiplicity	29
5 The Skew Product	
The Skew Product: Definition and its Measure Preserving Property	33
The Skew Product: Its Spectrum	34
6 A Theorem of Nelson and Parry	
Statement of the Theorem	37
Weak von Neumann Automorphisms and Hyperfinite Actions	37
The Cocycle $C(g, x)$	38
The Random Cocycle and the Main Theorem	39
Remarks	43

7	Probability Measures on the Circle Group	
	Continuous Probability Measures on S^1 : They are Dense G_δ	45
	Measures Orthogonal to a Given Measure	46
	Measures Singular Under Convolution And Folding	47
	Rigid Measures	48
8	Baire Category Theorems of Ergodic Theory	
	Isometries of $L^p(X, \mathcal{B}, m)$	51
	Strong Topology on Isometries	52
	Coarse and Uniform Topologies on $\mathcal{G}(m)$	52
	Baire Category of Classes of Unitary Operators	55
	Baire Category of Classes of Non-Singular Automorphisms	58
	Baire Category of Classes of Measure Preserving Automorphisms	58
	Baire Category and Joinings	59
9	Translations of Measures on the Circle	
	A Theorem of Weil and Mackey	63
	The Sets $A(\mu)$ and $H(\mu)$ and Their Topologies	65
	Groups Generated by Dense Subsets of $A(\mu)$; Their Properties	67
	Ergodic Measures on the Circle Group	68
	A Theorem on Marginal Measures	71
10	B. Host's Theorem	
	Pairwise Independent and Independent Joinings of Automorphisms	75
	B. Host's Theorem: The Statement	76
	Mixing Implies Multiple Mixing if the Spectrum is Singular	76
	B. Host's Theorem: The Proof	76
	An Improvement and an Application	81
11	L^∞ Eigenvalues of Non-Singular Automorphisms	
	The Group of Eigenvalues and Its Polish Topology	83
	Quasi-Invariance of the Spectrum	85
	The Group $e(T)$ is σ -Compact.	86
	The Group $e(T)$ is Saturated	87
12	Generalities on Systems of Imprimitivity	
	Spectral Measures and Group Actions	91
	Cocycles; Systems of Imprimitivity	94
	Irreducible Systems of Imprimitivity	95
	Transitive Systems	96
	Transitive Systems on \mathbb{R}	96

13 Dual Systems of Imprimitivity	99
Compact Group Rotations; Dual Systems of Imprimitivity	99
Irreducible Dual Systems; Examples	100
The Group of Quasi-Invariance; Its Topology	103
The Group of Quasi-Invariance; It is an Eigenvalue Group	104
Extensions of Cocycles	105
14 Saturated Subgroups of the Circle Group	109
Saturated Subgroups of S^1	109
Relation to Closures and Convex Hulls of Characters	111
σ -Compact Saturated Subgroups; H_2 Groups	114
15 Riesz Products As Spectral Measures	
Dissociated Trigonometric Polynomials	119
Classical Riesz Products and a Theorem of Peyrière	120
Riesz Products and Dynamics	122
Generalised Riesz Products	124
Maximal Spectral Types of Rank One Automorphisms	128
Examples and Remarks	131
The Non-Singular Case, Proof of Theorem 15.18, and Further Remarks	133
Rank One Automorphisms: Their Group of Eigenvalues	135
Preliminary Calculations	135
The Functions γ_k	136
The Eigenvalue Group: Osikawa Criterion	138
Restatement of Theorem 15.50.	139
The Eigenvalue Group: Structural Criterion.	140
An Expression for $\frac{d\sigma_\alpha}{d\sigma}$, $\alpha \in e(T)$	145
16 Additional Topics	
Bounded Functions with Maximal Spectral Type	149
A Result on Mixing	151
A Result On Multiplicity	153
Combinatorial and Probabilistic Lemmas	155
Rank One Automorphisms by Construction	159
Ornstein's Class of Rank One Automorphisms	160
Mixing Rank One Automorphisms	162
References	171
Index	179