

Contents

Chapter I. The Direct Methods in the Calculus of Variations	1
1. Lower Semi-Continuity	2
Degenerate Elliptic Equations, 4 — Minimal Partitioning Hypersurfaces, 5 — Minimal Hypersurfaces in Riemannian Manifolds, 7 — A General Lower Semi-Continuity Result, 8	
2. Constraints	13
Semi-Linear Elliptic Boundary Value Problems, 14 — Perron's Method in a Variational Guise, 16 — The Classical Plateau Problem, 19	
3. Compensated Compactness	25
Applications in Elasticity, 28 — Convergence Results for Nonlinear Elliptic Equations, 30	
4. The Concentration-Compactness Principle	34
Existence of Extremal Functions for Sobolev Embeddings, 40	
5. Ekeland's Variational Principle	47
Existence of Minimizers for Quasi-Convex Functionals, 51	
6. Duality	54
Hamiltonian Systems, 57 — Periodic Solutions of Nonlinear Wave-Equations, 61	
Chapter II. Minimax Methods	66
1. The Finite-Dimensional Case	66
2. The Palais-Smale Condition	69
3. A General Deformation Lemma	73
Pseudo-Gradient Flows on Banach Spaces, 73 — Pseudo-Gradient Flows on Manifolds, 77	

4. The Minimax Principle	79
Closed Geodesics on Spheres, 81	
5. Index Theory	86
Krasnoselskii Genus, 86 — Minimax Principles for Even Functionals, 88 — Applications to Semilinear Elliptic Problems, 90 — General Index Theories, 91 — Ljusternik-Schnirelman Category, 92 — A Geometrical S^1 -Index, 93 — Multiple Periodic Orbits of Hamiltonian Systems, 95	
6. The Mountain Pass Lemma and its Variants	100
Applications to Semilinear Elliptic Boundary Value Problems, 102 — The Symmetric Mountain Pass Lemma, 104 — Applications to Semilinear Equations with Symmetry, 108	
7. Perturbation Theory	110
Applications to Semilinear Elliptic Equations, 111	
8. Linking	116
Applications to Semilinear Elliptic Equations, 119 — Applications to Hamiltonian Systems, 121	
9. Critical Points of Mountain Pass Type	128
Multiple Solutions of Coercive Elliptic Problems, 132	
10. Non-Differentiable Functionals	135
11. Ljusternik-Schnirelman Theory on Convex Sets	147
Applications to Semilinear Elliptic Boundary Value Problems, 151	
 Chapter III. Limit Cases of the Palais-Smale Condition	154
1. Pohožaev's Non-Existence Result	155
2. The Brezis-Nirenberg Result	158
Constrained Minimization, 159 — The Unconstrained Case: Local Compactness, 160 — Multiple Solutions, 165	
3. The Effect of Topology	168
A Global Compactness Result, 169 — Positive Solutions on Annular-Shaped Regions, 175 — Remarks on the Yamabe Problem, 178	
4. The Dirichlet Problem for the Equation of Constant Mean Curvature	180
Small Solutions, 181 — The Volume Functional, 183 — Wente's Uniqueness Result, 185 — Local Compactness, 186 — Large Solutions, 188	

5. Harmonic Maps of Riemannian Surfaces	191
The Euler-Lagrange Equations for Harmonic Maps, 192 — The Homotopy Problem and its Functional Analytic Setting, 194 — Existence and Non-Existence Results, 195 — The Evolution of Harmonic Maps, 197	
Appendix A	211
Sobolev Spaces, 211 — Hölder Spaces, 212 — Imbedding Theorems, 212 — Density Theorem, 213 — Trace and Extension Theorems, 213 — Poincaré Inequality, 214	
Appendix B	216
Schauder Estimates, 216 — L^p -Theory, 216 — Weak Solutions, 217 — A Regularity Result, 217 — Maximum Principle, 219 — Weak Maximum Principle, 220 — Application, 221	
Appendix C	222
Fréchet Differentiability, 222 — Natural Growth Conditions, 224	
References	225
Index	243