

# Contents

## **A. The "classical" Plateau problem for disc-type minimal surfaces.**

### **I. Existence of a solution**

|                                                     |    |
|-----------------------------------------------------|----|
| 1. The parametric problem                           | 5  |
| 2. A variational principle                          | 7  |
| 3. The direct methods in the calculus of variations | 12 |
| 4. The Courant-Lebesgue Lemma and its consequences  | 16 |
| 5. Regularity                                       | 22 |
| Appendix                                            | 29 |

### **II. Unstable minimal surfaces**

|                                                                  |    |
|------------------------------------------------------------------|----|
| 1. Ljusternik-Schnirelman theory on convex sets in Banach spaces | 33 |
| 2. The mountain-pass lemma for minimal surfaces                  | 41 |
| 3. Morse theory on convex sets                                   | 52 |
| 4. Morse inequalities for minimal surfaces                       | 60 |
| 5. Regularity                                                    | 66 |
| 6. Historical remarks                                            | 78 |

## **B. Surfaces of prescribed constant mean curvature.**

### **III. The existence of surfaces of prescribed constant mean curvature spanning a Jordan curve in $\mathbb{R}^3$**

|                                |     |
|--------------------------------|-----|
| 1. The variational problem     | 91  |
| 2. The volume functional       | 94  |
| 3. "Small" solutions           | 100 |
| 4. Heinz' non-existence result | 104 |
| 5. Regularity                  | 105 |

### **IV. Unstable $H$ - surfaces**

|                                                                       |     |
|-----------------------------------------------------------------------|-----|
| 1. $H$ - extensions                                                   | 111 |
| 2. Ljusternik-Schnirelman and Morse theory for "small" $H$ - surfaces | 116 |
| 3. Large solutions to the Dirichlet Problem                           | 121 |
| 4. Large solutions to the Plateau problem: "Rellich's conjecture"     | 127 |

|            |     |
|------------|-----|
| References | 141 |
|------------|-----|