Contents

Chapter 1 A Brief Account of the Variational Principles of Classical Holonomic Dynamics

Preface

		/1
2.9	References	71
2.8	Other Variational Principles	69
2.7	Lagrangians with Vanishing Parameters	68
2.6	Partial Differential Equations	65
2.5	The Inverse Problem of the Calculus of Variations	60
2.4	Some Other Lagrangians	55
2.3	Quadratic Lagrangians for Systems with One Degree of Freedom	52
2.2	Lagrangians for Systems with One Degree of Freedom	48
2.1	Introduction	45
Cha	upter 2 Variational Principles and Lagrangians	
1.11	References	43
	Some Elementary Examples Involving Integral Variational Principles References	36
1 10	Dynamical Systems	34
1.9	Variational Principles Describing the Paths of Conservative	
1.8	Hamilton's Principle	26
1.7	A Brief Analysis of the Lagrangian Equations	15
1.6	Generalized Coordinates. Lagrangian Equations	12
1.5	The Lagrangian Equations with Multipliers	8
1.4	D'Alembert's Principle	7
1.3	Actual and Virtual Displacements	4
1.2	Constraints and the Forces of Constraint	1
1.1	Introduction	1

ix

Chapter 3 Conservation Laws

3.1	Introduction	74
3.2	Simultaneous and Nonsimultaneous Variations. Infinitesimal	7.5
3.3	Transformations The Condition of Invariance of Hamilton's Action Integral. Absolute a	75
3.3	Gauge Invariance	80
3.4	The Proof of Noether's Theorem. Conservation Laws	83
3.5	The Inertial Motion of a Dynamical System. Killing's Equations	85
3.6	The Generalized Killing Equations	87
3.7	Some Classical Conservation Laws of Dynamical Systems Completely	
	Described by a Lagrangian Function	91
	Examples of Conservation Laws of Dynamical Systems	97
3.9	Some Conservation Laws for the Kepler Problem	105
3.10	Inclusion of Generalized Nonconservative Forces in the Search for	
	Conservation Laws. D'Alembert's Principle	111
3.11	Inclusion of Nonsimultaneous Variations into the Central	
2 12	Lagrangian Equation	117
3.12	The Conditions for Existence of a Conserved Quantity. Conservation Le	
2 12	of Nonconservative Dynamical Systems The Generalized Killing Equations for Nonconservative Dynamical Systems	118 ems 120
	Conservation Laws of Nonconservative Systems Obtained by Means of	ins 120
3.17	Variational Principles with Noncommutative Variational Rules	121
3.15	Conservation Laws of Conservative and Nonconservative Dynamical	121
	Systems Obtained by Means of the Differential Variational Principles of	f
	Gauss and Jourdain	123
3.16	Jourdainian and Gaussian Nonsimultaneous Variations	127
3.17	The Invariance Condition of the Gauss Constraint	129
3.18	An Equivalent Transformation of Jourdain's Principle	132
3.19	The Conservation Laws of Schul'gin and Painlevé	133
3.20	Energy-Like Conservation Laws of Linear Nonconservative	
	Dynamical Systems	135
	Energy-Like Conservation Laws of Linear Dissipative Dynamical System	ns 140
	A Special Class of Conservation Laws	144
3.23	References	149
Cha	apter 4 A Study of the Motion of Conservative and	
	Nonconservative Dynamical Systems by Means	
	of Field Theory	
4.1	Introduction	152
4.2	Hamilton's Canonical Equations of Motion	153
4.3	Integration of Hamilton's Canonical Equations by Means of the	
	Hamilton-Jacobi Method	162
4.4	Separation of Variables in the Hamilton-Jacobi Equation	175
4.5	Application of the Hamilton-Jacobi Method to Linear Nonconservative	:
	Oscillatory Systems	180
4.6	A Field Method for Nonconservative Dynamical Systems	190

	·	ontents VII
4.7	The Complete Solutions of the Davis Field Facution and Their Davis	
4.7 4.8	The Complete Solutions of the Basic Field Equation and Their Proper	
4.9	The Single Solutions of the Basic Field Equation Illustrative Examples	202 202
	Applications of the Complete Solutions of the Basic Field Equation to	
4.10	Two-Point Boundary-value Problems	209
4.11	The Potential Method of Arzhanik'h for Nonconservative	20)
	Dynamical Systems	213
4.12	Applications of the Field Method to Nonlinear Vibration Problems	218
	A Linear Oscillator with Slowly Varying Frequency	235
4.14	References	238
Cha	upter 5 Variational Principles with Vanishing Parameter	`s
	and Their Applications	
	••	
5.1	Introduction	240
5.2	A Short Review of Some Variational Formulations Frequently Used in	
	Nonconservative Field Theory	241
5.3	The Variational Principle with Vanishing Parameter	248
5.4	Application of the Direct Method of Partial Integration to the Solution of Linear and Nonlinear Boundary-Value Problems	on 252
5.5	An Example: A Semi-Infinite Body with a Constant Heat Flux Input	252 253
5.6	A Semi-Infinite Body with an Arbitrary Heat Flux Input	260
5.7	The Temperature Distribution in a Body Whose End is Kept at Const.	
J.,	Temperature, Temperature-Dependent Thermophysical Coefficients	265
5.8	The Moment-Lagrangian Method	269
5.9	The Temperature Distribution in a Finite Rod with a Nonzero Initial	
	Temperature Distribution	272
5.10	The Temperature Distribution in a Noninsulated Solid	275
5.11	Applications to Laminar Boundary Layer Theory	276
5.12	Applications to Two-Dimensional Boundary Layer Flow of Incompres	ssible,
	Non-Newtonian Power-Law Fluids	287
5.13	A Variational Solution of the Rayleigh Problem for a Non-Newtonian	
	Power-Law Conducting Fluid	295
5.14	References	302
Ch.		
Cna	pter 6 Variational Principles with Uncommutative	
	Rules and Their Applications to	
	Nonconservative Phenomena	
6.1	Introduction	306
6.2	The Variational Principle with Uncommutative Rules	307
6.3	The Connection (Relation) between the Variational Principle with	307
	Uncommutative Rules and the Central Lagrangian Equation	309
6.4	The Bogoliubov-Krylov-Mitropolsky Method in Nonlinear Vibration	
	Analysis as a Variational Problem	314
6.5	Applications to Heat Conduction in Solids	317
6.6	References	330

viii Contents

Chapter 7 Applications of Gauss's Principle of Least Constraint to Nonconservative Phenomena

7.1	Introduction	332
7.2	Methods of Approximation Based on the Gauss Principle of	
	Least Constraint	333
7.3	Applications to Ordinary Differential Equations	340
7.4	Applications to Transient, Two-Dimensional, Nonlinear Heat Conduction	
	through Prism-Like Infinite Bodies with a Given Cross Section	344
7.5	Melting or Freezing of a Semi-Infinite Solid	348
7.6	A Semi-Infinite Solid with an Arbitrary Heat Flux Input: Gauss's Approach	353
7.7	A Nonconservative Convective Problem	357
7.8	References	360
	Author Index	363
	Index	367