
Contents

Preface	ix
Prologue: Why Wavelets?	xiii
1 Wavelets: A Brief Introduction	1
1.1 The Discrete Fourier Transform	1
1.2 The Haar System	7
Multiresolution Analysis	14
The Wavelet Representation	16
Goals of Multiresolution Analysis	22
1.3 Smoother Wavelet Bases	23
2 Basic Smoothing Techniques	29
2.1 Density Estimation	29
Histograms	31
Kernel Estimation	32
Orthogonal Series Estimation	35
2.2 Estimation of a Regression Function	38
Kernel Regression	39
Orthogonal Series Estimation	42
2.3 Kernel Representation of Orthogonal Series Estimators	45
3 Elementary Statistical Applications	49
3.1 Density Estimation	49
Haar-Based Histograms	49
Estimation with Smoother Wavelets	52
3.2 Nonparametric Regression	54
4 Wavelet Features and Examples	59
4.1 Wavelet Decomposition and Reconstruction	59
Two-Scale Relationships	60
The Decomposition Algorithm	62
The Reconstruction Algorithm	63
4.2 The Filter Representation	66

4.3	Time-Frequency Localization	69
	The Continuous Fourier Transform	69
	The Windowed Fourier Transform	72
	The Continuous Wavelet Transform	74
4.4	Examples of Wavelets and Their Constructions	79
	Orthogonal Wavelets	81
	Biorthogonal Wavelets	83
	Semiorthogonal Wavelets	87
5	Wavelet-based Diagnostics	89
5.1	Multiresolution Plots	89
5.2	Time-Scale Plots	92
5.3	Plotting Wavelet Coefficients	95
5.4	Other Plots for Data Analysis	100
6	Some Practical Issues	103
6.1	The Discrete Fourier Transform of Data	104
	The Fourier Transform of Sampled Signals	104
	The Fast Fourier Transform	105
6.2	The Wavelet Transform of Data	107
6.3	Wavelets on an Interval	110
	Periodic Boundary Handling	111
	Symmetric and Antisymmetric Boundary Handling	112
	Meyer Boundary Wavelets	113
	Orthogonal Wavelets on the Interval	114
6.4	When the Sample Size is Not a Power of Two	115
7	Other Applications	119
7.1	Selective Wavelet Reconstruction	119
	Wavelet Thresholding	124
	Spatial Adaptivity	126
	Global Thresholding	128
	Estimation of the Noise Level	131
7.2	More Density Estimation	132
7.3	Spectral Density Estimation	133
7.4	Detections of Jumps and Cusps	140
8	Data Adaptive Wavelet Thresholding	143
8.1	SURE Thresholding	144
8.2	Threshold Selection by Hypothesis Testing	149
	Recursive Testing	151
	Minimizing False Discovery	154
8.3	Cross-Validation Methods	156
8.4	Bayesian Methods	161

9	Generalizations and Extensions	167
9.1	Two-Dimensional Wavelets	167
9.2	Wavelet Packets	173
	Wavelet Packet Functions	174
	The Best Basis Algorithm	177
9.3	Translation Invariant Wavelet Smoothing	180
	Appendix	185
	References	191
	Glossary of Notation	199
	Glossary of Terms	201
	Index	205