

Table of Contents

Chapter One. Support Functionals.

1

The Bishop-Phelps theorem. . . James' characterization of weakly compact subsets of a Banach space. . . application of James' theorem to criterion for the reflexivity of the space of continuous linear operators.

Chapter Two. Convexity and Differentiability of Norms.

20

§1. Smoothness and Gateaux Differentiability of the Norm: support mappings. . . equivalence of smoothness at a point and Gateaux differentiability of the norm at a point. . . strict convexity. . . duality of strictly convex and smooth Banach spaces. . . James orthogonality. . . criterion for right and left uniqueness of orthogonality relations.

20

§2. Fréchet Differentiability and Local Uniform Convexity: characterization of Fréchet differentiability of norm at a point with norm-to-norm continuity of support mapping at the point. . . very smooth Banach spaces. . . density characters in very smooth Banach spaces. . . weak local uniform convexity and local uniform convexity. . . duality between local uniform convexity and Fréchet differentiability and between weak local uniform convexity and very smooth spaces.

29

§3. Convexity and Smoothness in Higher Duals: discussion of the deterioration of convexity and smoothness of norms in higher duals of non-reflexive Banach spaces.

33

§4. Uniform Smoothness, Uniform Convexity and Their Duality: equivalence of uniform smoothness, uniformly Frechet differentiable norm and norm-to-norm uniform continuity of support maps. . . duality between uniformly smooth and uniformly convex Banach spaces. . . reflexivity of uniformly convex spaces.

35

- §5. Convexity, normal structure and fixed point theorems: normal structure. . . non-expansive maps. . . weakly compact convex subsets having normal structure have the fixed point property with respect to the class of non-expansive maps. . . commutative families of non-expansive maps and existence of common fixed points. 38
- Chapter Three. Uniformly convex and uniformly smooth Banach spaces. 54
- §1. The uniform convexity of the $L_p(\mu)$ -spaces ($1 < p < \infty$). 54
- §2. Unconditionally Convergent Series in Uniformly Convex Banach Spaces. 57
- §3. The Day-Nordlander theorem. 60
- §4. The modulus of smoothness and the divergence of series in Banach spaces. 62
- §5. The moduli of convexity of the L_p spaces. 68
- §6. Bases in uniformly convex and uniformly smooth Banach spaces (the theorem of V. I. Gurarii and N. I. Gurarii). 73
- §7. The Banach-Saks property: Kakutani's theorem showing all uniformly convex Banach spaces have the Banach-Saks property. . . spaces with Banach-Saks property are reflexive. 78

Chapter Four. The classical renorming theorems. 94

- §1. Day's norm on $c_0(\Gamma)$: J. Rainwater's proof that M. M. Day's norm on $c_0(\Gamma)$ is locally uniformly convex. 94

§2. General facts about renorming: criteria for existence of equivalent strictly convex norm. . . Troyanski's sufficient condition for existence of an equivalent locally uniformly convex norm. . . criteria for equivalent norm on a dual space to be a dual norm. 100

§3. Asplund's averaging technique: the method of E. Asplund to average two norms, one of a given degree of convexity, the other with a dual norm of a given degree of convexity, to obtain a third (equivalent) norm with both features. 106

§4. The Kadec-Klee-Asplund renorming theorem: if X is a Banach space with separable dual then X can be renormed so that X^* is smooth and locally uniformly convex. 113

§5. Possible and impossible renormings of ℓ_∞ : ℓ_∞ while strictly convexifiable is not smoothable nor weakly locally uniformly convexifiable. . . . if Γ is an uncountable set then $\ell_\infty(\Gamma)$ is not strictly convexifiable. 120

Chapter Five: Weakly compactly generated Banach spaces. 128

§1. Fundamental Lemmas: the construction of "long sequences" of continuous linear projections in weakly compactly generated spaces. 128

§2. Basic Results in WCG Banach spaces: existence of continuous injection of WCG Banach space into $c_0(\Gamma)$. . . resulting renorming theorems. . . weakly compact subsets of Banach spaces always "live" in $c_0(\Gamma)$ -spaces. . . Eberlein compacts. . . Ω is Eberlein compact if and only if $C(\Omega)$ is WCG. . . the dual ball of a WCG Banach space is weak-star sequentially compact. . . separable subspaces of WCG spaces are contained in complemented separable subspaces. . . operators on Grothendieck spaces... discrete generation of WCG Banach spaces. . . the Johnson-Lindenstrauss stability criteria for WCG Banach spaces. 143

§3. Rosenthal's Topological Characterization of Eberlein Compacts: Grothendieck's criteria for weak compactness in $C(\Omega)$. . . $C(\Omega)$ is a WCG 156

Banach algebra if and only if it is a WCG Banach space. . . Ω is Eberlein compact if and only if Ω admits a sequence (G_n) of point-finite families of open- \mathfrak{F}_G sets such that $\bigcup_n G_n$ is separating.

§4. The Factorization of Weakly Compact Linear Operators: the remarkable factorization theorem of W. J. Davis, T. Figiel, W. B. Johnson and A. Pelczynski with applications. 160

§5. Trujanski's theorem: every WCG Banach space has an equivalent locally uniformly convex norm. . . reflexive Banach spaces always have an equivalent norm; it and its dual norm are locally uniformly convex and Fréchet differentiable. 164

§6. Operators Attaining Their Norm. The Bishop-Phelps Property: the collection of continuous linear operators between Banach spaces which achieve their maximum norm on the weak-star closure in X^{**} of a closed bounded convex set of X is dense in the space of all operators. . . every weakly compact, convex subset of a Banach space is the closed convex hull of its strongly exposed points. 167

§7. The Friedland-John-Zizler Theorem: If X is a WCG Banach space and Y is a closed linear subspace of X with an equivalent very smooth norm then Y is WCG. . . in a WCG Banach space with equivalent Fréchet differentiable norm all closed linear subspaces are WCG. 173

§8. A Theorem of W. B. Johnson and J. Lindenstrauss: if X is a Banach space with WCG dual and X embeds in a WCG Banach space, then X is WCG. 177

§9. The John-Zizler Renorming Theorem: if X and X^* are WCG, then X can be renormed in a locally uniformly convex, Fréchet differentiable manner where the dual norm on X^* is also locally uniformly convex and smooth. 185

§10. Counterexamples to General Stability Results for WCG Banach Spaces: a discussion and description of the Johnson-Lindenstrauss example of a non-WCG Banach space with WCG dual and of Rosenthal's 189

example of a non-WCG closed linear subspace of an $L_1(\mu)$ -space for finite μ .

Chapter Six: The Radon-Nikodým Theorem for Vector Measures. 199

§1. The Bochner Integral: review of notions of strong measurability, 199
Pettis integrability and Bochner integrability.

§2. Dentability and Rieffel's Criteria: the notion of dentability; 203
M. A. Rieffel's characterization of differentiable measures. . . examples
of non-differentiable vector measures.

§3. The Davis-Huff-Maynard-Phelps Theorem: equivalence of the Radon- 213
Nikodym property with dentability of bounded sets. . . stability criteria
for Radon-Nikodym property. . . renorming spaces with Radon-Nikodym property.

§4. The Dunford-Pettis Theorem: the classical result of N. Dunford 221
and B. J. Pettis to the effect that separable dual spaces possess the
Radon-Nikodym property. . . dual subspaces of WCG spaces have the
Radon-Nikodym property. . . the Dunford-Pettis-Phillips theorem on
representability of weakly compact operators on L_1 . . . weakly compact
subsets of $L_\infty(\mu)$, μ -finite, are separable.

§5. A Lindenstrauss Result: The Krein-Milman property. . . the 230
Radon-Nikodym property implies the Krein-Milman property.

§6. The Huff-Morris-Stegall Theorem: in dual spaces the Radon- 233
Nikodym property, the Krein-Milman property and the imbeddability of
separable subspaces into separable duals are equivalent. . . further
stability results for the Radon-Nikodym property. . . if X^* possesses
the Radon-Nikodym property then bounded sequences in X have weak
Cauchy subsequences.

§7. Edgar's Theorem: a Choquet type theorem for separable closed 246
bounded convex subsets of a Banach space with the Radon-Nikodym
property.

§8. A Theorem of R. R. Phelps: a lemma of E. Bishop. . . Phelps' 252
characterization of the Radon-Nikodym property as that of closed bounded
convex sets being the closed convex hull of strongly exposed points.